Learning and Collusion in New Markets with Uncertain Entry Costs*

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Abstract

This paper analyzes an entry timing game with uncertain entry costs. Two firms receive costless signals about the cost of a new project and decide when to invest. We characterize the equilibrium of the investment timing game with private and public signals. We show that competition leads the two firms to invest too early and analyze two collusion schemes, one in which one firm pays the other to stay out of the market, and one, in which this buy-out is mediated by a third party. We characterize conditions under which the efficient outcome can be implemented in both collusion schemes.
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1 Introduction

When firms contemplate entering a new geographical market, they typically conduct market research to learn about the demand and they investigate various production and distribution alternatives. The final decision of whether to enter the foreign market is only taken after months of investigation; and it is not uncommon that two rival firms join forces by creating a joint subsidiary in order to access foreign markets. Similarly, when firms are engaged in a race to obtain an innovation, they often start building small prototypes, or running small scale experiments before investing in a large scale research project. Once a firm has started developing a new product, the race will often be concluded with a merger, with one of the two firms acquiring the research output of the other before the new product reaches the market. Sometimes, the investment decision is mediated by an outside agency, like a venture capitalist or a granting agency.

In this paper, our objective is to better understand the interplay between experimentation with the goal of acquiring knowledge about the cost of entry and preemption in entry timing games, and to study collusive mechanisms between two firms engaged in the development of new products or the access to new markets. The questions we aim at answering are: Do firms invest too early or too late? How does the fact that signals are public or private affect the entry timing decisions? When do simple compensating payment allow firms to achieve the collusive outcome? Which share of the surplus should accrue to the two firms in the collusive transfer scheme? When is the optimal time to implement cooperation?

To this end, we study investment decisions by two firms which compete to enter a new market or develop a new product. Ex-ante, the firms are uncertain about the cost of the investment. They gradually acquire signals about their entry cost through research and experimentation. (We consider here the case of *private values*, where the entry costs are independently distributed across firms.) Upon observing their signals, and forging beliefs about the signals received by their competitors, firms decide when to enter. If both firms enter, they collect duopoly profits on the market; if only one firm enters, it will receive monopoly profits. We suppose that firms make positive profits as duopolists only when their entry cost is low, and make positive expected profits as monopolists when they have not yet learned their costs. We compute the cooperative outcome, where the two firms choose entry timing in order to maximize joint profits, and the outcome of the noncooperative game of entry timing played by the two firms both when the signals are public and private. We analyze how the two firms can implement a collusive transfer scheme by which the firm which has entered the market compensates the other firm for not entering and under which conditions they can implement a collusive mechanism, in which an outside party has the power to determine the firms' joint investment decisions.

Our first set of results deals with the firms' incentives to learn their cost of entry in a non-cooperative setting. We find that competition leads firms to invest excessively early, and that excess momentum is higher when signals are private than public. We consider situations of *project selection*, where it is optimal for the two firms to wait until they learn that one of the two projects is profitable before entering the market. For sufficiently low expected entry costs, competition leads the two firms to invest immediately in order to preempt entry by the rival firm. Furthermore, there exists an intermediate range of the expected entry costs, for which firms choose to wait when signals are public but preempt at a finite time when signals are private. With private signals, firms do not observe whether their competitor has abandoned the race or not. As time passes, firms become more and more convinced that the other firm has dropped from the race (no news is good news). Hence, at some finite date, firms become sufficiently confident that the other firm will not enter the market, and choose to enter before they learn their entry cost. This equilibrium is reminiscent of the preemption equilibrium in the innovation race studied by Fudenberg and Tirole (1985) and Grossman and Shapiro (1987). However, in our model, preemption occurs due to the endogenous dynamics of beliefs, whereas in their model, preemption results from the exogenous dynamics of the innovation cost.

Our second set of results deals with collusive schemes, which allow the firms to the collusive outcome. Competing firms face three sources of inefficiency: (i) market competition, (ii) duplication of entry costs and (iii) excess momentum in market entry. We consider two schemes by which firms can seek to reach the collusive outcome. First, we study compensating payments which are paid by the investing firm after it has invested to the other firm so that it stays out of the market. Second, we examine a compensating mechanism where an outside party simultaneously chooses which firm invests and chooses the transfers to be paid. With the first scheme, in which the designer cannot determine the investment decisions, we show that collusion is possible only when a firm enters the market sufficiently early. After a finite date, collusion becomes impossible as the active firm becomes convinced that its rival has dropped from the market and is unwilling to compensate it at a level which would prevent entry. We also show that in order to achieve efficient entry timing decisions, the monopoly surplus should be shared between the active and inactive firms in an equitable fashion. The share of the active firm should be large enough to give it an incentive to invest immediately after it learns its cost. The share of the inactive firm should be large enough so that firms have no incentive to enter early in order to preempt their rival. With the second scheme, in which the designer also determines the investment decisions, we show that it is possible to implement the collusive outcome at any point in time without payments to the inactive firm as long as the expected payoff in the outcome in which firms wait to learn their cost is sufficiently high. If it is not sufficiently high, the implementation of the first best requires again a sharing of the surplus from collusion between the active and the inactive firm.

Our analysis thus sheds light on situations of project selection, where two independent firms run parallel research programs and a third party can enforce a cooperative scheme to prevent inefficiencies. The third party can for instance be a venture capitalist or a granting agency running competing research project, the editor of an academic journal or organizer of a scientific conference who discovers that two teams of scientists are working on the same problem. Our analysis suggests that selection should neither occur too early (before the profitabilities of the projects are known), nor too late (when the firms have become very optimistic about their prospects given that the other firm has not entered). It also shows that the share of the surplus transferred to the firm which is not selected should neither be too large (in which case the selected firm may have an incentive to delay the research project) nor too small (the higher the payoff transferred to the firm which is not selected, the smaller the gap between the payoffs of the leading and trailing firms, which reduces inefficiencies due to excess momentum.)

Our analysis is rooted in the literature on patent races in continuous time pioneered by Reinganum (1982) and Harris and Vickers (1985). The first extensions of patent races allowing for symmetric uncertainty are due to Spatt and Sterbenz (1985), Harris and Vickers (1987) and Choi (1991). Models of learning in continuous time with public information have been studied by Keller and Rady (1999) and Keller, Cripps and Rady (2005) in the more complex environment of bandit problems. Rosenberg, Solan and Vieille (2007) and Murto and Välimäki (2011) analyze general stopping games with common values where players' payoffs does not depend on the actions of other players. Similarly, Chamley and Gale (1994), Lambrecht and Perraudin (2003) and Decamps and Mariotti (2004) model market entry with private information about the common value of market entry within a real options framework. By contrast, we consider the setting of private values but consider strategic interaction between the players after entry.

The model of preemption we consider is formally identical to Fudenberg and Tirole's (1985) model of technology adoption with preemption. Innovation timing games which can result either in preemption or in waiting games have been studied by Katz and Shapiro (1987). Hoppe and Lehmann-Grube (2005) propose a general method for analyzing innovation timing games. Fudenberg and Tirole's (1985) model has been extended by Weeds (2002) and Mason and Weeds (2010) to allow for stochastic values of the technology. However, none of these models allows for private information. The closest papers to ours are the recent papers by Hopenhayn and Squintani (2011) on preemption games with private information and Moscarini and Squintani (2010) on patent races with private information. Moscarini and Squintani (2010) analyze a common values problem, where agents learn about the common arrival rate of the innovation, whereas we analyze a private values problem in which agents learn their individual market entry cost. Accordingly, our model displays very different results. Even though Hopenhayn and Squintani's (2011) model is more general than ours in many aspects, it only covers situations where agents receive positive information over time. In our model, research teams may either receive positive or negative signals about the profitability of the research project. This impacts the results so that the insights of Hopenhayn and Squintani (2011) do not directly apply. Cooperation among research teams with private information has been studied in a mechanism design context by Gandal and Scotchmer (1993). Goldfain and Kovac (2007) analyze the optimal design of contracts by a venture capitalist running two parallel projects. Gordon (2011) and Akcigit and Liu (2013) study patent races with private signals, focussing on the incentives to disclose information to competitors. More generally, Athey and Segal (2013) study efficient dynamic mechanisms. Their team mechanism does not apply directly to our framework due to our assumption of post-entry competition.

The rest of the paper is organized as follows. We introduce the model and describe the collusive benchmark in Section 2. Section 3 contains our core analysis of entry timing games with public and private signals. Section 4 discusses the design of schemes that allow the firms to attain the collusive outcome. Conclusions and directions for future research are given in Section 5. All proofs and derivations are collected in the Appendix.

2 The Model

2.1 Firms, new markets and entry costs

The model specification follows closely Murto and Välimäki (2011). We consider a model with discrete time. We let $t = 0, 1, .., \infty$ denote the periods in the game. The discount factor per period is $\delta = e^{-r\Delta}$ where Δ is the period length and $r > 0$ the pure rate of time preference. At the beginning of the game, nature chooses the entry cost of the two firms, $\theta_i \in \{\overline{\theta}, \underline{\theta}\}$, but firms do not know their entry cost. We consider a model of *private values* where costs are randomly and independently chosen for the two firms, and for simplicity, assume that high and low costs are equiprobable.[2] The expected value of the entry cost is thus given by

$$\widetilde{\theta} = \frac{\overline{\theta} + \underline{\theta}}{2}.$$

The information about private entry costs arrives gradually through the game. During the experimentation phase, each firm receives every period a signal $\xi \in \{0, 1, 2\}$. We assume that $\Pr(\xi = 0|\theta = \underline{\theta}) = \Pr(\xi = 2|\theta = \overline{\theta}) = \lambda\Delta$, $\Pr(\xi = 1|\theta = \underline{\theta}) = \Pr(\xi = 1|\theta = \overline{\theta}) = 1 - \lambda\Delta$ and $\Pr(\xi = 2|\theta = \underline{\theta}) = \Pr(\xi = 0|\theta = \overline{\theta}) = 0$ where $\lambda > 0$ is a commonly known parameter, and the period length Δ is small enough so that $\lambda\Delta < 1$. Hence, with probability $1-\lambda\Delta$, the firm does not learn its type during the period, and with probability $\lambda\Delta$, the firm receives a perfect signal about its entry cost. Signals are independent across periods and across players (conditional on the types), and are privately observed by each firm. No payoff is collected by the firms during the experimentation phase.

At each period t, both firms simultaneously make a binary choice, $e_i^t \in \{0, 1\}$. If $e_i^t = 1$, firm i enters the market, pays its entry cost θ, stops the experimentation phase and starts collecting profits. The profits collected by the firm depends on the entry of the other firm. When both firms are present on the market, they each collect a duopoly payoff of $v_d\Delta$ per period. When a single monopolistic firm operates on the market, it collects the monopoly profit $v_m\Delta$ every period. We assume that $v_m > 2v_d$.

[2] The analysis would not change if we assumed different probabilities for the high and low costs.

We suppose that high cost firms never have an incentive to invest, even if they receive monopoly profit. Low cost firms always have an incentive to invest even if they receive duopoly profit. When entry cost remains unknown, firms have an incentive to invest as monopolists but not as duopolists. Formally, we define the discounted sum of duopoly and monopoly profits as:

$$\pi_d = \frac{\Delta v_d}{1 - e^{-r\Delta}}, \pi_m = \frac{\Delta v_m}{1 - e^{-r\Delta}},$$

and assume:

Assumption 1

$$\underline{\theta} \leq \pi_d \leq \widetilde{\theta} \leq \pi_m \leq \overline{\theta}. \tag{1}$$

2.2 Entry timing and strategies

In our model, every firm privately observes its signals during the experimentation phase, and the entry decisions are public. Notice that the profit of a firm only depends on the entry decision of the other firm (and not on its entry cost), so that, after a firm has entered, whether or not its entry cost is revealed to the other firm is immaterial. Hence, the model we consider is in a very strong sense a private values model, where the payoff of a firm does not depend on the type of the other firm. A firm's strategy specifies, for any given private history up to t, its entry decision at period t. We consider *perfect equilibrium strategies* which maximize the firm's expected discounted payoff after every possible history.

Given Assumption 1, it is a dominant strategy for a high cost firm not to invest. In addition, we show below that it is a dominant strategy for a firm which learns that its cost is low to invest immediately. Hence, the only relevant decision is whether or not a firm which has not yet learned its type chooses to invest.

Lemma 1 *It is a dominant strategy for a firm which learns that it has a low cost at period t to invest immediately.*

Lemma 1 is a very intuitive result, but still requires a proof. In general, the other firm may adopt a mixed strategy, choosing to invest with positive probability when it has not yet learned its cost, or to delay investment after learning that its cost is low. In order to show that a firm with a low cost has an incentive to invest immediately, we not only observe that the firm loses a positive profit of $\pi_d - \underline{\theta}$ (or $\pi_m - \underline{\theta}$) for one period by delaying investment, but also note that, by delaying investment, the firm may facilitate the entry of the other firm, resulting in a loss from monopoly profit to duopoly profit. Given Lemma 1, *the only relevant investment strategy is the strategy chosen by a firm which has not yet learned its cost.* From now on, we will characterize equilibrium by focusing on this strategic choice.

2.3 Collusive benchmark

In this section, we abstract from competition between the two firms and characterize the optimal investment strategy chosen by a monopolist who has access to both technologies. Notice that there are three possible strategies:

1. Invest immediately, resulting in a payoff $V_{E0} = \pi_m - \widetilde{\theta}$.

2. Experiment with both technologies, and invest after receiving the first low signal. Let t denote the time at which the first firm learns its type, either the type is $\underline{\theta}$ and the firm invests or it is $\overline{\theta}$ and the firm continues experimenting with the remaining project. At time $\tau \geq t+1$ it learns the type of the second project and invests if it has cost $\underline{\theta}$. Finally, we consider the case where both firms learn the types simultaneously and invest with probability 3/4. Hence, the expected collusive payoff is given by

$$\begin{aligned} V_{EL} &= \sum_{t=1}^{\infty}(1-\lambda\Delta)^{2(t-1)}(\lambda\Delta(1-\lambda\Delta)\delta^t[(\pi_m - \underline{\theta}) \\ &\quad + \sum_{\tau=t+1}^{\infty}(1-\lambda\Delta)^{\tau-t-1}\frac{\lambda\Delta}{2}\delta^{\tau-t}(\pi_m-\underline{\theta})] + \frac{3\lambda^2\Delta^2}{4}\delta^t(\pi_M - \underline{\theta})) \\ &= \frac{\delta\lambda\Delta(\pi_m - \underline{\theta})}{1-\delta(1-\lambda\Delta)^2}[1 - \frac{\lambda\Delta}{4} + \frac{\delta\lambda\Delta}{2(1-\delta(1-\lambda\Delta))}] \end{aligned}$$

3. Experiment with both technologies and invest after receiving the first signal. Following the same lines as above, we note that investment will occur as soon as one of the firms learns its type, resulting in a payoff of $\pi_m - \underline{\theta}$ if the project has low cost and $\pi_m - \widetilde{\theta}$ if it has high cost and the firm invests in the other project. The expected collusive

parameter region	optimal cooperative choice
$\widetilde{\theta} < \pi_m - V_{E2}$	invest immediately
$\pi_m - V_{E2} \leq \widetilde{\theta} < \pi_m - V_{E1}$	invest immediately after receiving the first signal
$\pi_m - V_{E1} \leq \widetilde{\theta}$	invest only after receiving a low-cost signal

Table 1: Optimal collusive choice

payoff is given by

$$V_{E2} = \sum_{t=1}^{\infty}(1-\lambda\Delta)^{2(t-1)}\delta^t(1-\lambda\Delta)\lambda\Delta(2\pi_m - \underline{\theta} - \widetilde{\theta}) + \frac{3(\lambda\Delta)^2}{4}(\pi_m - \underline{\theta})$$
$$= \frac{\delta\lambda\Delta}{1-\delta(1-\lambda\Delta)^2}((1-\lambda\Delta)(\pi_m - \widetilde{\theta}) + (1 - \frac{\lambda\Delta}{4}(\pi_m - \underline{\theta})).$$

If a monopolist experiments with a single technology, it obtains an expected payoff

$$V_{E1} = \frac{\delta\lambda\Delta(\pi_m - \underline{\theta})}{2(1-\delta(1-\lambda\Delta))}.$$

Letting the length of every period Δ go to zero, we easily compute :

$$\begin{aligned} V_{EL} &= \frac{\lambda}{2\lambda + r}(\pi_m - \underline{\theta})(1 + \frac{\lambda}{2(\lambda + r)}), \\ V_{E2} &= \frac{2\lambda}{2\lambda + r}(\pi_m - \frac{\underline{\theta} + \widetilde{\theta}}{2}), \\ V_{E1} &= \frac{\lambda}{2(\lambda + r)}(\pi_m - \underline{\theta}) \end{aligned}$$

Notice that, when $\Delta \to 0$, $V_{EL} - V_{E2}$ converges to $V_{E1} - (\pi_m - \widetilde{\theta})$ and furthermore that $V_{E2} \geq V_{E1}$. Hence we can summarize the optimal collusive strategy in Table 1, distinguishing different régimes as a function of the average fixed cost $\widetilde{\theta}$. For low $\widetilde{\theta}$, it is not worth experimenting and firms choose one project at random. For intermediate values of $\widetilde{\theta}$ it is worth experimenting with two but projects but not with one and the monopolist invests immediately after receiving the first signal. For high values of $\widetilde{\theta}$ it is worth experimenting with only one project and the monopolist only invests after having learned that one of the technologies has a low cost.

3 Entry timing

We now analyze the game played by two competing firms. We first compute the profits of the leader and follower firms. Suppose that one firm (the leader) invests first. The second firm (the follower) will only follow suit if it learns that its cost is low. Hence the expected value of the follower is given by

$$V_F = \frac{(\pi_d - \underline{\theta})\delta\lambda\Delta}{2(1-\delta(1-\lambda\Delta))},$$

and the expected value of the leader gross of the fixed cost is given by

$$V_L = \pi_m - \frac{(\pi_m - \pi_d)\delta\lambda\Delta}{2(1-\delta(1-\lambda\Delta))}.$$

We easily check that

$$V_L - V_F = \pi_m - V_{E1}.$$

and compute the values V_F and V_L as $\Delta \to 0$ as:

$$\begin{aligned} V_F &= \frac{\lambda}{2(\lambda + r)}(\pi_d - \underline{\theta}), \\ V_L &= \pi_m - \frac{\lambda}{2(\lambda + r)}(\pi_m - \pi_d). \end{aligned}$$

3.1 Entry timing with public signals

In this subsection, we suppose that the signals received by the two firms during the experimentation phase are *public*. If a firm observes that the other firm has drawn a high cost, it will either invest immediately, and obtain $\pi_m - \widetilde{\theta}$ or wait until it observes a signal on its cost and obtain V_{E1}. The optimal choice depends on the comparison between $\pi_m - \widetilde{\theta}$ and V_{E1}. If a firm observes that the other firm has drawn a low cost (and invested along the equilibrium path by Lemma 1), the optimal strategy is to wait and only invest if the firm receives a signal that its cost is low. Finally, as long as the other firm has not received the signal about its cost, the optimal strategy depends on the values of the parameters. If $\pi_m - \widetilde{\theta} > V_{E1}$, the game is a game of *preemption* and, as Δ goes to zero, the two firms will rush to invest at the beginning of the game. If, on the other hand, $\pi_m - \widetilde{\theta} < V_{E1}$, the game is a *waiting* game, and the firms will wait until they receive a low cost signal before investing. Proposition 1 summarizes this result.

Proposition 1 *In the entry timing game with public information as $\Delta \to 0$, if $\pi_m - \widetilde{\theta} > V_{E1}$, preemption occurs, firms invest immediately after the other firm has dropped out, and invest with positive probability at every date $t = 0, 1, 2, \ldots$. If $\pi_m - \widetilde{\theta} < V_{E1}$, firms do not invest until they learn that their cost is low.*

Comparing the equilibrium behavior of the two firms with the collusive benchmark, we observe that there is a parameter region ($\pi_m - V_{E2} \leq \widetilde{\theta} < \pi_m - V_{E1}$) where firms prefer to wait in the collusive benchmark but invest immediately in the noncooperative game. Hence, competition results in excess momentum, and the firms invest too early with respect to the collusive benchmark.

3.2 Entry timing with private signals

When signals are private, firms do not learn the cost of their competitor. Beliefs evolve over time. At any date, T, we denote by $\gamma_T(\theta)$ the belief held by a firm about the cost of its rival, conditional on the event that the rival has not invested. By Lemma 1, in equilibrium, because firms which learn that their cost is low invest immediately, $\gamma_T(\underline{\theta}) = 0$. We denote by $G(T)$ the probability with which a firm, which has not learned its cost, has invested at or before date T. With these notations in hand, we compute the beliefs

$$\begin{aligned}
\gamma_T(\overline{\theta}) &= \frac{\sum_{t=1}^{\frac{T}{\Delta}} (1-\lambda\Delta)^{t-1} \frac{\lambda\Delta}{2} [1 - G(\Delta(t-1))]}{\sum_{t=1}^{\frac{T}{\Delta}} (1-\lambda\Delta)^{t-1} \frac{\lambda\Delta}{2} [1 - G(\Delta(t-1))] + \sum_{t=1}^{\frac{T}{\Delta}} (1-\lambda\Delta)^{t} [1 - G(\Delta t)]}, \\
\gamma_T(\widetilde{\theta}) &= \frac{\sum_{t=1}^{\frac{T}{\Delta}} (1-\lambda\Delta)^{t} [1 - G(\Delta t)]}{\sum_{t=1}^{\frac{T}{\Delta}} (1-\lambda\Delta)^{t-1} \frac{\lambda\Delta}{2} [1 - G(\Delta(t-1))] + \sum_{t=1}^{\frac{T}{\Delta}} (1-\lambda\Delta)^{t} [1 - G(\Delta t)]}.
\end{aligned}$$

As $\Delta \to 0$, for any $T < T'$, if $G(T') < 1$, $G(T) = 0$. Hence, as $\Delta \to 0$, beliefs converge to

$$\begin{aligned}
\gamma_T(\overline{\theta}) &= \frac{1 - e^{-\lambda T}}{1 + e^{-\lambda T}}, \\
\gamma_T(\widetilde{\theta}) &= \frac{2e^{-\lambda T}}{1 + e^{-\lambda T}}.
\end{aligned}$$

It is easy to check that the belief that the other firm has learned that it has a high cost, $\gamma_T(\overline{\theta})$, increases over time. The expected profit of a firm which is the first to invest at date T now depends on time and is given by

$$V_L(T) = \gamma_T(\overline{\theta})\pi_m + \gamma_T(\widetilde{\theta})V_L.$$

Because $\gamma_T(\overline{\theta})$ is increasing over time, the value of the leader is also *increasing*. No news is good news: as time passes, each firm becomes more convinced that the other firm has received a negative signal, and becomes more optimistic about its own prospects. The value of the leader increases from V_L at $T = 0$ to π_m when T goes to infinity. The value of the follower, V_F, remains independent of time. Figures 1, 2 and 3 illustrate the three possible régimes, ranking the values of the leader and the follower, as a function of the parameters of the model.

Cases 1 and 3 correspond to the preemption and waiting cases in the timing game with public signals. Case 2 exploits the fact that beliefs evolve over time, and describes a new situation where preemption occurs at some finite date $\widetilde{T}$. In this case, the entry timing game is formally identical to the innovation game studied by Fudenberg and Tirole (1985). The expected payoff of the leading firm is initially lower than the expected payoff of the following firm, but is increasing over time and eventually becomes higher than the payoff of the following firm. As in Fudenberg and Tirole (1985), the unique subgame perfect equilibrium results in rent equalization: the leader invests exactly at the date where the expected payoffs of the leader and follower coincide, $V_L(\widetilde{T}) - \widetilde{\theta} = V_F$. Formally,

Theorem 1 *In the entry timing game with private signals, if $\pi_m - \widetilde{\theta} > V_{E_1}$, preemption occurs at the beginning of the game and both firms invest with positive probability at date 0. If $V_{E1} \geq \pi_m - \widetilde{\theta} \geq V_F$, in a symmetric equilibrium, rents between the leader and the follower are equalized and each firm invests with probability $\frac{1}{2}$ at date $\widetilde{T}$ such that: $V_L(\widetilde{T}) - \widetilde{\theta} = V_F$. If $V_F > \pi_m - \widetilde{\theta}$, firms do not invest unless they learn that their cost is low.*

Theorem 1 shows that excess momentum is higher with private signals than with public signals. In one configuration of the parameters, when $V_{E1} \geq \pi_m - \widetilde{\theta} \geq V_F$, firms wait to learn their costs in the collusive benchmark and when signals are public, but invest at finite date $\widetilde{T}$ when signals are private. This result stands in contrast to Hopenhayn and Squintani (2011) who show that preemption is stronger with public signals than with private signals. This difference is easily explained. In Hopenhayn and Squintani (2011), new information can only signal an improvement in the competitive situation of the firm, so that competition is fiercer when information is public. In our model, private information can only signal a degradation

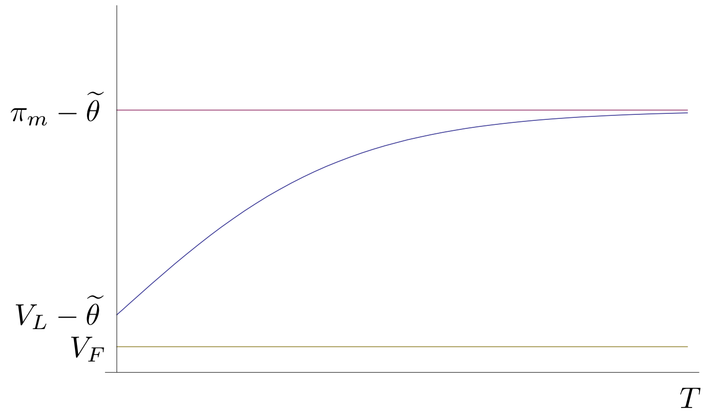

Figure 1: Case 1: $\pi_m - \widetilde{\theta} > V_{E_1}$

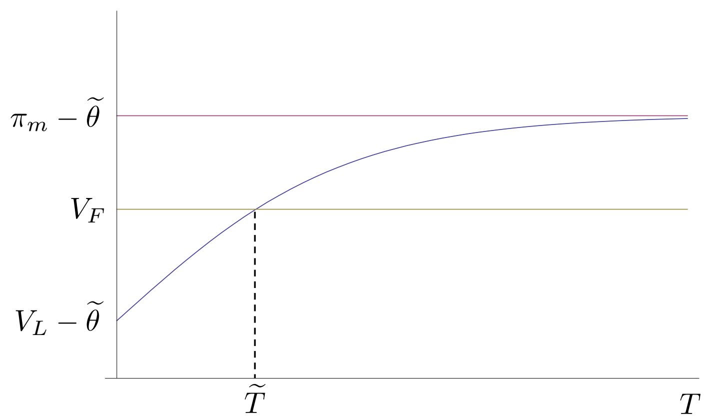

Figure 2: Case 2: $V_{E_1} \geq \pi_m - \widetilde{\theta} \geq V_F$

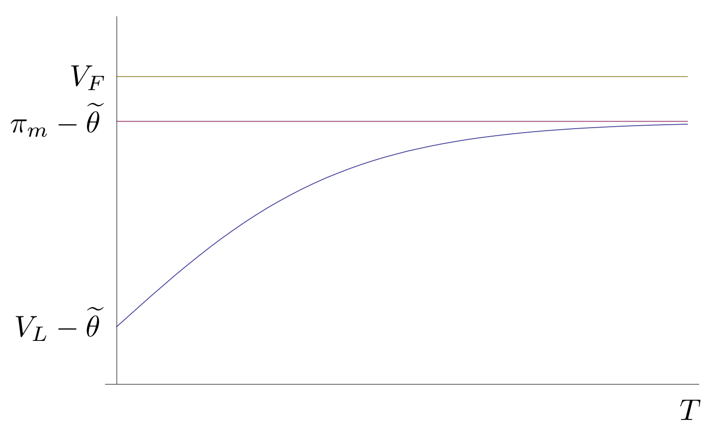

Figure 3: Case 3: $V_F > \pi_m - \widetilde{\theta}$

parameter	comparative static
π_m	$-$
π_d	$+$
$\overline{\theta}$	$+$
$\underline{\theta}$	$+$
r	$-$
λ	$+/-$

Table 2: PREEMPTION DATE $\tilde{T}$ – COMPARATIVE STATICS

in the competitive situation of the firm, so that competition is fiercer when information is private.

We now focus on Case 2 and perform a comparative static analysis of the effect of changes in the parameters of the model on the preemption date $\widetilde{T}$. The preemption date is implicitly defined as the unique solution to the equation:

$$V_L\left(T\right) - \widetilde{\theta} - V_F = 0. \tag{2}$$

By implicit differentiation of equation (2) when $\Delta \to 0$, we obtain the comparative statics displayed in Table 2.

The effects of the model's parameters on the preemption date, $\widetilde{T}$, are intuitive, except for the intensity of the arrival of the signal, λ. Changes in the signal arrival rate, λ, have ambiguous effects. An increase in the arrival rate accelerates the process by which a firm learns its cost, increasing the value of the follower:

$$\frac{dV_F}{d\lambda} = \frac{r}{2\left(\lambda + r\right)^2}\left(\pi_d - \underline{\theta}\right) > 0,$$

and decreasing the value of the leader at time zero

$$\frac{dV_L}{d\lambda} = -\frac{r}{2\left(\lambda + r\right)^2}\left(\pi_m - \pi_d\right) < 0.$$

In addition, an increase in λ increases the speed at which a firm updates its belief about its opponent. Hence, the rate at which $V_L(T)$ increases is higher and

$$\frac{dV_L(T)}{d\lambda} = \left(-\frac{r}{2(\lambda + r)^2}\gamma_T(\widetilde{\theta}) - \frac{d\gamma_T(\widetilde{\theta})}{d\lambda}\frac{\lambda}{2(\lambda + r)}\right)(\pi_m - \pi_d),$$

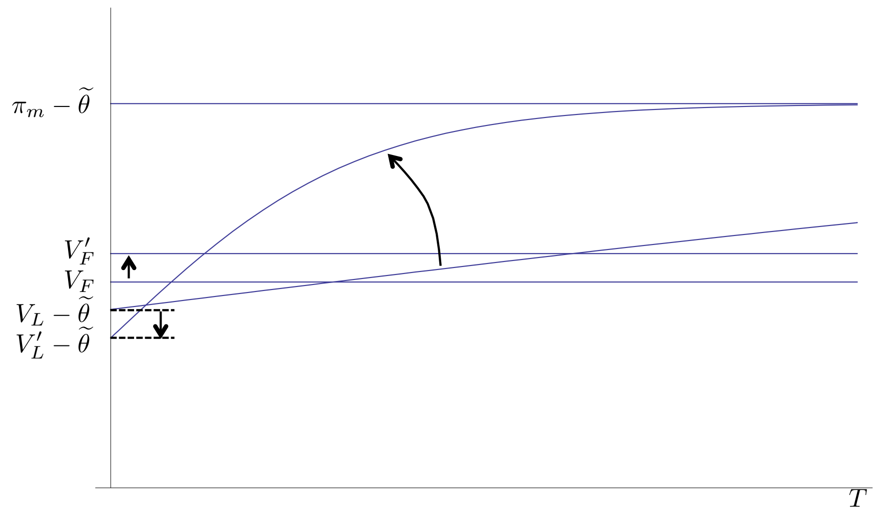

Figure 4: Effect of an increase in λ on $V_L(T)$ and V_F.

where $\frac{d\gamma_T(\widetilde{\theta})}{d\lambda} < 0$.

Figure 4 shows the effects of an increase in λ on V_F and $V_L(T)$. The effect of a change in λ on the preemption date $\tilde{T}$ is non-monotonic. If λ is low and $\tilde{T}$ is low, an increase in λ will mostly have the effect of increasing V_F and reducing V_L, resulting in an increase in the preemption date. If, on the other hand, λ is high and $\tilde{T}$ is high, an increase in λ will mostly have the effect of increasing $V_L(T)$, reducing the preemption date. This non-monotonicity is illustrated in Figure 5 which shows how $\tilde{T}$ varies with λ when $\pi_m = 0.7$, $\pi_d = 0.3$, $\overline{\theta} = 0.8$, $\underline{\theta} = 0.2$, and $r = 0.05$.

3.3 Efficiency comparison

We now compare the joint profits of the two firms in the collusive benchmark, the equilibrium of the noncooperative game with public signals and with private signals. We distinguish between four parameter regions, depending on the magnitude of the expected entry cost $\widetilde{\theta}$:

1. $\widetilde{\theta} < \pi_m - V_{E2}$: immediate entry in the cooperative regime, and preemption at zero in both competitive regimes;

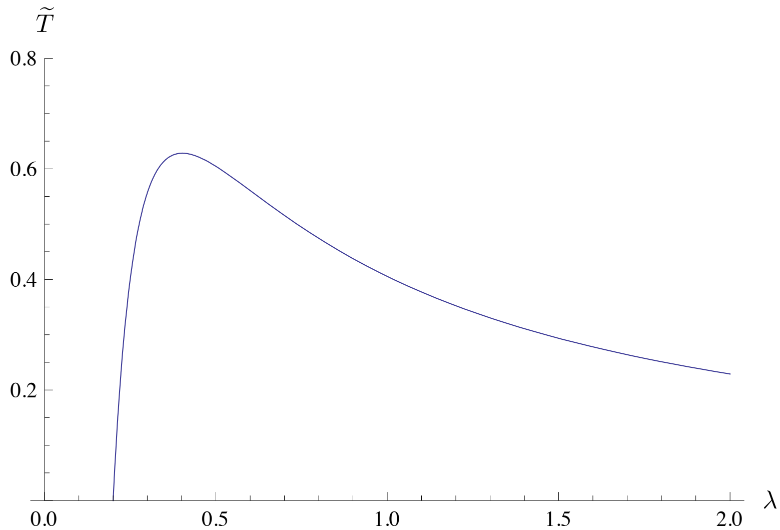

Figure 5: $\widetilde{T}$ AS A FUNCTION OF λ FOR $\pi_m = 0.7$, $\pi_d = 0.3$, $\overline{\theta} = 0.8$, $\underline{\theta} = 0.2$, AND $r = 0.05$.

2. $\pi_m - V_{E2} \leq \widetilde{\theta} < \pi_m - V_{E1}$: delayed entry in the cooperative regime (firms wait for one signal), and preemption at zero in both competitive regimes;

3. $\pi_m - V_{E1} \leq \widetilde{\theta} < \pi_m - V_F$: delayed entry in the cooperative regime (firms wait for up to two signals) and in the competitive regime with public signals, preemption at finite time $\widetilde{T}$ in the competitive regime with private signals;

4. $\pi_m - V_F \leq \widetilde{\theta}$: delayed entry in the cooperative regime (firms wait for up to two signals) and in both competitive regimes.

We focus on the case $\Delta \to 0$ and define the industry profits when both firms delay their entry until they learn that their cost is low as:

$$V_S = \frac{\lambda}{2\lambda + r}\left[(\pi_m - \underline{\theta}) + \frac{\lambda}{2(\lambda + r)}(2\pi_d - 2\underline{\theta})\right]$$

and the industry profits with preemption at finite time $\widetilde{T}$ as:

$$\begin{aligned} V_P &= \left(1 - e^{-(2\lambda+r)\widetilde{T}}\right)\frac{\lambda}{2\lambda + r}\left[(\pi_m - \underline{\theta}) + \frac{\lambda}{2(\lambda + r)}(2\pi_d - 2\underline{\theta})\right] \\ &+ e^{-(\lambda+r)\widetilde{T}}\left(1 - e^{-\lambda\widetilde{T}}\right)\frac{\lambda}{2(\lambda + r)}(\pi_m - \underline{\theta}) \\ &+ e^{-(2\lambda+r)\widetilde{T}} 2\frac{\lambda}{2(\lambda + r)}(\pi_d - \underline{\theta}) \end{aligned} \tag{3}$$

It is easy to check that $V_{E2} > V_S > V_P > 2V_F$ and $\pi_m - \widetilde{\theta} > 2V_F$. Table 3 lists the joint profits under the three regimes in the four configurations of parameters.

parameter region	cooperative	public	private
$\widetilde{\theta} < \pi_m - V_{E2}$	$\pi_m - \widetilde{\theta}$	$2V_F$	$2V_F$
$\pi_m - V_{E2} \leq \widetilde{\theta} < \pi_m - V_{E1}$	V_{EL}	$2V_F$	$2V_F$
$\pi_m - V_{E1} \leq \widetilde{\theta} < \pi_m - V_F$	V_{E2}	V_S	V_P
$\pi_m - V_F \leq \widetilde{\theta}$	V_{E2}	V_S	V_S

Table 3: Efficiency comparisons

Table 3 shows that joint profits are always higher when signals are public rather than private. Simple derivations also show that all values V_{E1}, V_{E2}, V_F, V_S are strictly increasing in λ, and numerical computations suggest that V_P is increasing in λ as well.

Table 3 illustrates three sources of inefficiency due to competition. First, by competing on the market, the firms forgo the benefits of market monopolization – the difference between monopoly profits, π_m and the sum of duopoly profits, $2\pi_d$. Second the firms pay twice the entry cost θ, whereas in the collusive benchmark, only one firm invests. Finally competition results in excess momentum, making firms enter the market before they learn their cost, whereas in the collusive benchmark, they prefer to wait until they learn their cost before entering.

4 Collusion and compensating payments

In this section, we analyze the design of schemes which allow the two firms to attain the collusive outcome. We suppose that signals are private and that excess momentum occurs, i.e. the parameters satisfy $\pi_m - V_{E2} \leq \widetilde{\theta} \leq \pi_m - V_F$. We consider two different settings. We first analyze *compensating payments* which are paid by the investing firm *after it has invested* in order to prevent the other firm from investing. We then consider a *collusive mechanism* where an outside party simultaneously makes the investment decision for the two firms and chooses the transfers to be paid. In both situations, our objective is to characterize conditions under which the optimal investment decision can be implemented. Notice that, because investment decisions are not made by the mechanism designer in the compensating payments scheme, the compensating payments environment is more constrained than the collusive mechanism environment, and implementation of the efficient outcome is more difficult.

4.1 Compensating payments

In this subsection, we assume that $\Delta \to 0$, and compute directly the asymptotic values of profits, when the time steps become negligible. Suppose that the leader invests at date T, and considers compensating the follower for not entering the market. As the follower's type is unknown to the leader, payments to the followers of types $\widetilde{\theta}$ and $\overline{\theta}$ must be equal. Let $U_L(T)$ and $U_L(T)$ denote the utilities of the leader and follower after compensation. By individual rationality, these payoffs must be higher than the payoffs obtained in the non-cooperative entry game.

$$U_L(T) \geq V_L(T), \tag{4}$$
$$U_F(T) \geq V_F. \tag{5}$$

Budget balance implies that the sum of utilities received by the leader and follower are equal to the monopoly profit:

$$U_L(T) + U_F(T) = \pi_m. \tag{6}$$

Given inequalities (4) and (5) and equality (6), a necessary condition for the existence of a budget balanced, individually rational and incentive compatible transfer scheme is thus

$$\pi_m \geq V_L(T) + V_F.$$

As $V_L(T)$ is increasing, $V_L(0) < \pi_m - V_F$ and $V_L(\infty) > \pi_m - V_F$, there exists a unique date T^*, such that *no budget balanced individually rational compensating payments exist if the first firm enters the market at date $T \geq T^*$.* This remark captures the following simple intuition. As time passes, firms become more optimistic about their prospects. If a firm enters at a late date, it will expect the other firm to have dropped and will not be willing to compensate the other firm at the level V_F, which is the minimal level that a firm which has not yet learned its cost is willing to accept to drop from the market. This remark also shows that *there is no efficient, budget balanced and individually rational collusive mechanism.* To see this, consider a realization of the signals where no firm has learned its cost before T^*.

Either the mechanism prescribes that one of the firm invests before T^*, and the mechanism is inefficient because it will result in a high cost firm investing with positive probability, or the mechanism prescribes to wait until one of the firm has learned it has a low cost, and the mechanism is inefficient because there is no budget balanced, individually rational compensating payment which prevents the other firm from entering the race.

The latest point at which firms can collude, T^*, is implicitly determined by

$$\pi_m = V_L(T^*) + V_F \tag{7}$$

Table 4 shows the other comparative statics of changes in parameters on the date T^*. Notice that a change in the Poisson arrival rate λ, has a clear negative effect on T^*. When firms learn their costs more quickly, beliefs evolve faster, and the last date at which collusion may occur is reduced.

parameter	comparative static
π_m	$+$
π_d	$-$
$\overline{\theta}$	0
$\underline{\theta}$	$+$
r	0
λ	$-$

Table 4: Latest date for cooperation T^* – Comparative statics

We now consider the following problem: How should compensating payments be designed in order to guarantee that, whenever one firm learns that it has a low cost before T^*, it is chosen to be the only firm operating on the market?

Proposition 2 *A differentiable compensating payment scheme $U_F(T)$ implements the cooperative benchmark when a firm learns that it has a low cost before T^* if and only if for all $T < T^*$,*

$$\pi_m - \widetilde{\theta} < 2U_F(T) < \frac{2r+\lambda}{r+\lambda}\pi_m + \frac{U_F'(T)}{r+\lambda}.$$

Proposition 2 shows that efficient compensating payment schemes must be designed to satisfy two requirements. First, the payment to the follower must be large enough to prevent early entry by firms which have not yet learned their costs. Second, the payment to the follower should not be too large, in order to give incentives to a firm which learns that its cost is low to enter immediately. These two requirements provide an upper and a lower bound on the expected payoffs of the follower and leader firm and show that the cooperative surplus must be shared in a balanced way between the two firms.

In order to provide additional intuition, we specialize the model by assuming that the compensating payment scheme assigns a fixed bargaining power to the leader and the follower, so that

$$U_L(T) = V_L(T) + \alpha(\pi_m - V_L(T) - V_F), \tag{8}$$
$$U_F(T) = V_F + (1-\alpha)(\pi_m - V_L(T) - V_F). \tag{9}$$

We observe that $U_L(T)$ is increasing and $U_F(T)$ decreasing over time. Figure 6 displays these profits for $\alpha = 0$ and $\alpha = 1$. It illustrates three aspects of the model. First, it displays the T^*, for which $\pi_m - V_L(T^*) - V_F = 0$. Second, Figure 6 shows that payoffs are independent of time if $\alpha = 1$, that is, if all of the bargaining power is given to the leader. In this case, the follower receives his outside utility, $U_F = V_F$, and the leader receives all surplus plus his outside utility, $U_L(T) = \pi_m - V_F$. Third, Figure 6 shows that the gap between the payoff of the leader and follower is increasing in α.

Using (8) and (9), Proposition 2 imposes two restrictions on the share of the surplus that accrues to the leader, α. On the one hand, to prevent firms to invest early when they have not yet learned their cost, α has to be sufficiently low:

$$\alpha < \frac{\pi_m + \widetilde{\theta} - V_L(T)}{2(\pi_m - V_L(T) - V_F)}. \tag{10}$$

On the other hand, to give a firm that learns that it has low cost incentives to enter immediately, α must be sufficiently high:

$$\frac{2(\pi_m - V_L(T)) + \frac{V_L'(T)}{r+\lambda} - \frac{2r+\lambda}{r+\lambda}\pi_m}{2(\pi_m - V_L(T) - V_F) + \frac{V_L'(T)}{r+\lambda}} < \alpha. \tag{11}$$

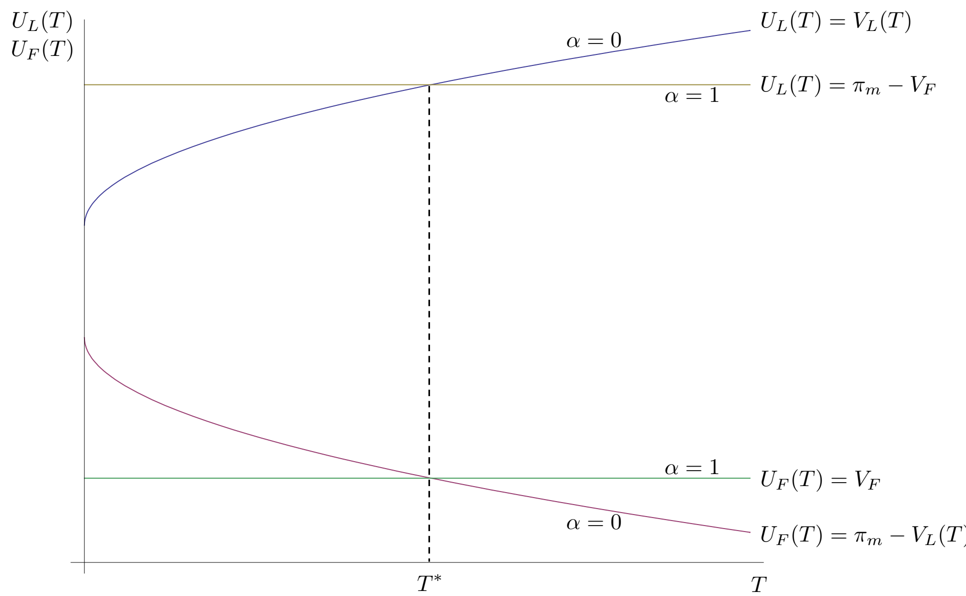

Figure 6: Expected utilities with compensating payments

Define

$$\overline{\alpha} \equiv \min\left\{\frac{\pi_m + \widetilde{\theta} - V_L(T)}{2\left(\pi_m - V_L(T) - V_F\right)}, 1\right\}$$

and

$$\underline{\alpha} \equiv \max\left\{0, \frac{2\left(\pi_m - V_L(T)\right) + \frac{V_L'(T)}{r+\lambda} - \frac{2r+\lambda}{r+\lambda}\pi_m}{2\left(\pi_m - V_L(T) - V_F\right) + \frac{V_L'(T)}{r+\lambda}}\right\}.$$

Corollary 1 *A compensating payment scheme that assigns a fixed bargaining power to the leader and the follower firm, so that a share α of the surplus from cooperation accrues to the leader, implements the collusive outcome when a firm learns that it has a low cost before T^* if and only if for all $T < T^*$,$\alpha \in [\underline{\alpha}, \overline{\alpha}]$.*

A necessary condition for implementation of the collusive outcome is that $0 \leq \underline{\alpha} \leq \overline{\alpha} \leq 1$, which is guaranteed if the following conditions on the parameters hold:

$$2V_F \geq \pi_m - \widetilde{\theta}, \tag{12}$$

$$2V_F + 2(1-\alpha)(\pi_m - V_L(0) - V_F) \leq \frac{2r+\lambda}{r+\lambda}\pi_m - \frac{V_L'(0)}{r+\lambda}. \tag{13}$$

Notice that if condition (12) fails, early preemption will occur before T^*. However, the value of α can be designed in order to delay entry of firms which have not yet learned their costs as far as possible. By reducing α, and giving a larger share of the surplus to the follower, the mechanism designer reduces incentives to preempt and delays inefficient entry of firms on the market.[3] The optimal compensating payment mechanism is then given by the *lowest value of* α for which condition (13) holds.

4.2 Collusive mechanism

We now suppose that the mechanism designer can choose to assign the investment to one of the two firms, and we analyze collusion as a standard mechanism design problem. As firms' types are persistent and are revealed through time, the problem is a problem of dynamic mechanism design and we consider a dynamic sequence of mechanisms M_T, for $T = \Delta, 2\Delta, ..., t\Delta, ..$ where at each date T, the planner asks the two firms to report their types $(\widehat{\theta}_1, \widehat{\theta}_2)$ where $\widehat{\theta}_i \in \{\underline{\theta}, \widetilde{\theta}, \overline{\theta}\}$ for $i = 1, 2$. Messages sent by firm i are *not observed* by firm j and are not revealed by the mechanism designer. At date T, the planner has access to a history of messages $h_T = ((\widehat{\theta}_1, \widehat{\theta}_2)^1, (\widehat{\theta}_1, \widehat{\theta}_2)^2, ..., (\widehat{\theta}_1, \widehat{\theta}_2)^{T-1})$. In order to support the efficient outcome, we assume that the designer chooses to invest *as soon as she learns that one of the firms has a low cost.* Hence, the game stops either when the designer learns that one of the firms has a low cost, or when the two firms have reported high costs.

A history is *inconsistent* if one of the firms reports $\widehat{\theta}_i = \overline{\theta}$ at a date T and $\widehat{\theta}_i = \widetilde{\theta}$ or $\widehat{\theta}_i = \underline{\theta}$ at a date $T' > T$. If the planner observes an inconsistent history, we assume that she punishes both players by imposing high penalties, so that we can disregard inconsistent histories. The only public signal is the planner's decision to invest. Given that we rule out inconsistent histories, at the time of investment, the only relevant information that the planner can extract from history h^T is calendar time T and the revealed types of the firms at T, $(\widehat{\theta}_1, \widehat{\theta}_2)$. Hence, without loss of generality, we assume that transfers at T only depend on T and the types revealed at T and not on the entire history of messages. Notice that, given our assumptions, if firm j has sent the message $\overline{\theta}$ to the planner, this message will only be revealed to firm i if it invests later in the game. Formally, at any date T,

[3] The fact that giving a prize to the loser of a contest may be efficient, as it reduces the gap between the winner and the loser and minimizes wasteful expenditures, has long been noted in the literature on contests. See for example Moldovanu and Sela (2001).

- If $\widehat{\theta}_i \neq \underline{\theta}, \widehat{\theta}_j \neq \underline{\theta}$, the planner does not select any firm to invest

- If $\widehat{\theta}_i = \underline{\theta}, \widehat{\theta}_j \neq \underline{\theta}$ at time T, the planner selects firm i to invest. Firm i pays a tax $\chi_T(\widehat{\theta}_j)$ and firm j receives a subsidy $\sigma_T(\widehat{\theta}_j)$.

- If $\widehat{\theta}_i = \widehat{\theta}_j = \underline{\theta}$, each firm is chosen to invest with probability $\frac{1}{2}$. The firm which invests pays a tax $\chi_T(\underline{\theta})$ whereas the firm which does not invest receives a subsidy $\sigma_T(\underline{\theta})$. We assume that the two firms are treated symmetrically, so that $\pi_m - \underline{\theta} - \chi_T(\underline{\theta}) = \sigma_T(\underline{\theta}) = \frac{\pi_m - \underline{\theta}}{2}$.

Individual rationality We denote the expected values from participating in this mechanism for a firm of type θ at time T as $EV_T(\theta)$. The expected utilities are computed as follows,

$$EV_T(\underline{\theta}) = \frac{\lambda\Delta}{2}\frac{\pi_m - \underline{\theta}}{2} + (1 - \frac{\lambda\Delta}{2})(\pi_m - \underline{\theta} - (\gamma_T(\overline{\theta})\chi_T(\overline{\theta}) + \gamma_T(\widetilde{\theta})\chi_T(\widetilde{\theta}))), \tag{14}$$

$$EV_T(\overline{\theta}) = \gamma_T(\widetilde{\theta})\sum_{t=0}^{\infty}(1-\lambda\Delta)^{t-1}\delta^t\frac{\lambda\Delta}{2}\sigma_{T+\Delta t}(\overline{\theta}), \tag{15}$$

$$\begin{aligned} EV_T(\widetilde{\theta}) = {} & \gamma_T(\overline{\theta})\sum_{t=0}^{\infty}(1-\lambda\Delta)^{2(t-1)}\delta^t\frac{\lambda\Delta}{2}(\pi_m - \underline{\theta} - \chi_{T+\Delta t}(\overline{\theta})) \\ & +\gamma_T(\widetilde{\theta})[\sum_{t=0}^{\infty}(1-\lambda\Delta)^{2(t-1)}\delta^t\frac{\lambda\Delta}{2}(\pi_m - \underline{\theta} - \chi_{T+\Delta t}(\widetilde{\theta}) + \sigma_{T+\Delta t}(\widetilde{\theta})) \\ & + \sum_{\tau=0}^{\infty}(1-\lambda\Delta)^{\tau-1}\delta^\tau\frac{\lambda\Delta}{2}(\pi_m - \underline{\theta} - \chi_{T+\Delta(t+\tau)}(\widetilde{\theta}) + \sigma_{T+\Delta(t+\tau)}(\overline{\theta}))]. \end{aligned} \tag{16}$$

Individual rationality is satisfied if, at every date T, every type of firm prefers to abide by the mechanism than to play the market entry game. In order to compute the expected payoffs in the market entry game, we need to specify the beliefs that firm i holds on the type of firm j, *given that firm j has chosen not to participate in the mechanism.* As we consider a situation where both firms choose to participate, these beliefs are off the equilibrium path, and we are free to specify them. We will suppose that, upon observing that a firm refuses to participate, the other firm initially believes that it is a low cost firm – and hence refrains from entering the market. However, if a firm has observed the other firm not participating, and then choosing not to invest, the firm keeps her initial beliefs γ_T. This assumption of conservative belief updating, which follows Laffont and Martimort (1997), allows us to compute the expected equilibrium payoffs of a firm playing the market entry game at time T as follows.

By playing the market entry game, a firm with low cost obtains an expected payoff of

$$EU_T(\underline{\theta}) = (\frac{\lambda\Delta}{2}\pi_d + (1 - \frac{\lambda\Delta}{2})V_L(T)) - \underline{\theta}$$

and a firm with high cost an expected payoff of

$$EU_T(\overline{\theta}) = 0$$

In order to compute the expected payoff of a firm which has not yet learned its cost, recall that, at $\widetilde{T}$, one of the firms will invest. Hence, if $T < \widetilde{T}$, the expected profit is is given by:

$$\begin{aligned} EU_T(\widetilde{\theta}) = &\sum_{t=1}^{\frac{\tilde{T}-T}{\Delta}}(1-\lambda\Delta)^{2(t-1)}\delta^t[\frac{(\lambda\Delta)^2}{4}(\pi_d - \underline{\theta}) + (1 - \frac{\lambda\Delta}{2})\frac{\lambda\Delta}{2}(V_L(T+\Delta t) - \underline{\theta} + V_F)] \\ &+ (1-\lambda\Delta)^{2\frac{\tilde{T}-T}{\Delta}}\delta^{\widetilde{T}-T}V_F. \end{aligned}$$

If $T > \tilde{T}$, following the proof of Theorem 1, the game becomes a game of preemption, and both firms invest with positive probability $p(T)$ in equilibrium, resulting in an equilibrium payoff

$$EU_T(\widetilde{\theta}) = \gamma_T(\widetilde{\theta})p(T)(\pi_d - \widetilde{\theta}) + \gamma_T(\widetilde{\theta})(1-p(T))(V_L(T) - \widetilde{\theta}) + \gamma_T(\overline{\theta})(\pi_m - \widetilde{\theta}),$$

where

$$p(T) = \frac{V_L(T) - V_F - \widetilde{\theta}}{\gamma_T(\widetilde{\theta})(V_L - \pi_d)}.$$

Hence, we get the individual rationality constraints

$$EV_T(\underline{\theta}) \geq EU_T(\underline{\theta}), \qquad (IR(\underline{\theta}))$$
$$EV_T(\widetilde{\theta}) \geq EU_T(\widetilde{\theta}), \qquad (IR(\widetilde{\theta}))$$
$$EV_T(\overline{\theta}) \geq EU_T(\overline{\theta}). \qquad (IR(\overline{\theta}))$$

Incentive compatibility *High cost firms* A high cost firm never has an incentive to report that its cost is low as $\pi_m - \overline{\theta} - \chi_T(\theta) \leq \pi_m - \overline{\theta} < 0$. In order to analyze the high cost firm's incentive to misreport a type $\widetilde{\theta}$, we apply the one-step deviation principle, and derive the condition under which firm $\overline{\theta}$ has no incentive to report $\widetilde{\theta}$ for one period, and then switch back to $\overline{\theta}$. By misreporting its type in that way, the high cost firm can only gain $\sigma_T(\widetilde{\theta}) - \sigma_T(\overline{\theta})$ with probability $\frac{\lambda\Delta}{2}$ in period T (when the other firm reports a low cost), and obtains the same continuation value $EV_{T+1}(\overline{\theta})$ from period $T+1$ on. Hence, the incentive compatibility condition is given by:

$$\sigma_T(\overline{\theta}) \geq \sigma_T(\widetilde{\theta}) \qquad (IC(\overline{\theta} \to \widetilde{\theta}))$$

Low cost firms If a low cost firm reports that it has a high cost, it will obtain an expected payoff of $EV_T(\overline{\theta})$, so the first IC constraint is:

$$EV_T(\underline{\theta}) \geq EV_T(\overline{\theta}). \qquad (IC(\underline{\theta} \to \overline{\theta}))$$

If a low cost firm pretends it has not yet learned its cost at period T and then reports a low cost at time $T+1$, it will obtain an expected payoff:

$$\frac{\lambda\Delta}{2}\frac{\pi_m - \underline{\theta}}{2} + (1 - \frac{\lambda\Delta}{2}\delta[\frac{\lambda\Delta}{2}\sigma_{T+1}(\underline{\theta}) + (1 - \frac{\lambda\Delta}{2})\pi_m - \underline{\theta} - (\gamma_{T+1}(\overline{\theta})\chi_{T+1}(\overline{\theta}) + \gamma_{T+1}(\widetilde{\theta})\chi_{T+1}(\widetilde{\theta}))],$$

whereas by announcing $\underline{\theta}$ immediately, it obtains an expected payoff of $EV_T(\underline{\theta})$. Hence the incentive compatibility condition reads:

$$\begin{aligned} EV_T(\underline{\theta}) \geq & \frac{\lambda\Delta}{2}\sigma_T(\widetilde{\theta}) + (1 - \frac{\lambda\Delta}{2}\delta[\frac{\lambda\Delta}{2}\sigma_{T+1}(\underline{\theta}) \\ & + (1 - \frac{\lambda\Delta}{2})\pi_m - \underline{\theta} - (\gamma_{T+1}(\overline{\theta})\chi_{T+1}(\overline{\theta}) + \gamma_{T+1}(\widetilde{\theta})\chi_{T+1}(\widetilde{\theta}))]. \qquad (IC(\underline{\theta} \to \widetilde{\theta})) \end{aligned}$$

Firms ignoring their costs: If a firm which has not learned its cost reports a high cost $\overline{\theta}$, it will obtain a payoff of $EV_T(\overline{\theta})$. Hence, the first incentive compatibility constraint is given by:

$$EV_T(\widetilde{\theta}) \geq EV_T(\overline{\theta}). \qquad (IC(\widetilde{\theta} \to \overline{\theta}))$$

Similarly, by announcing a low cost, the firm obtains a payoff of $EV_T(\underline{\theta})$ so the second incentive compatibility constraint is:

$$EV_T(\widetilde{\theta}) \geq (1 - \frac{\lambda\Delta}{2})[\pi_m - \widetilde{\theta} - (\gamma_T(\overline{\theta})\chi_T(\overline{\theta}) + \gamma_T(\widetilde{\theta})\chi_T(\widetilde{\theta}))] + \frac{\lambda\Delta}{2}[\frac{\pi_m - \widetilde{\theta}}{2}]. \quad (IC(\widetilde{\theta} \to \underline{\theta}))$$

Budget balance Ex post budget balance requires that, at any period T and for any type θ, the transfers add up to zero, so that

$$\sigma_T(\theta) = \chi_T(\theta) \tag{BB}$$

Imposing the constraints We now characterize the binding and non-binding constraints in the mechanism design problem.

Remark 1 *The individual rationality constraint for the high cost firm, $IR(\overline{\theta})$ and incentive compatibility constraint $IC(\overline{\theta} \to \underline{\theta})$ are not binding. Hence, the only binding constraints for the high cost firm is $IC(\overline{\theta} \to \widetilde{\theta})$. This implies that we can assume $\sigma_T(\overline{\theta}) = \sigma_T(\widetilde{\theta})$ for all T.*

Proof: It is easy to check that the constraints $IR(\overline{\theta})$ and $IC(\overline{\theta} \to \underline{\theta})$ are not binding. But then the only binding constraint is $IC(\overline{\theta} \to \widetilde{\theta})$, which implies that $\sigma_T(\overline{\theta}) \geq \sigma_T(\widetilde{\theta})$. Now choosing $\sigma_T(\overline{\theta}) > \sigma_T(\widetilde{\theta})$ makes the incentive and individual rationality constraints of agents of types $\underline{\theta}$ and $\widetilde{\theta}$ harder to satisfy, and does not relax the problem faced by the high cost firm, as the constraints are not binding. Hence, we can assume $\sigma_T(\overline{\theta}) = \sigma_T(\widetilde{\theta}) = \sigma_T$ for all T in order to characterize the efficient mechanism.

Remark 2 *If the incentive compatibility constraint $IC(\widetilde{\theta} \to \overline{\theta})$ is satisfied, so is the incentive compatibility constraint $IC(\underline{\theta} \to \overline{\theta})$.*

Proof: By a repeated application of the incentive compatibility constraint $IC(\underline{\theta} \to \widetilde{\theta})$, we observe that

$$EV_T(\underline{\theta}) \geq EV_T(\widetilde{\theta}).$$

Hence, whenever $EV_T(\widetilde{\theta}) \geq EV_T(\overline{\theta})$, we also have $EV_T(\underline{\theta}) \geq EV_T(\overline{\theta})$.

Remark 3 *Suppose that the mechanism does not extract resources from the firms, i.e. $\sigma_T(\theta) \geq \chi_T(\theta)$ for all T. Then the individual rationality constraint $IR(\widetilde{\theta})$ is not binding.*

Proof: Because the mechanism selects the efficient outcome x, for any pair (θ_i, θ_j) at date T, and the sum of transfers is positive, we have:

$$V_i^T(x, \theta_i, \theta_j) + V_j^T(x, \theta_i, \theta_j) \geq V_i^T(y, \theta_i, \theta_j) + V_j^T(y, \theta_i, \theta_j)$$

for any other investment choice y. When $\theta_i = \theta_j = \widetilde{\theta}$, the two firms have identical information about their types and the type of their competitor. Hence, integrating over time and taking expectations with respect to the same distribution, we have

$$EV_i^T(\widetilde{\theta}) + EV_j^T(\widetilde{\theta}) \geq EU_i^T(\widetilde{\theta}) + EU_j^T(\widetilde{\theta}),$$

so that $EV_T(\widetilde{\theta}) \geq EU_T(\widetilde{\theta})$.

We can thus focus on the following five constraints: $IR(\underline{\theta}), IC(\overline{\theta} \to \widetilde{\theta}), IC(\widetilde{\theta} \to \overline{\theta}), IC(\underline{\theta} \to \widetilde{\theta}), IC(\widetilde{\theta} \to \underline{\theta})$. Taking $\Delta \to 0$,[4] and using $IC(\overline{\theta} \to \widetilde{\theta})$, that is, $\sigma_T(\overline{\theta}) = \sigma_T(\widetilde{\theta}) = \sigma_T$, these constraints can be written as

$$\pi_m - \sigma_T \geq V_L(T), \tag{$IR(\underline{\theta})$}$$

$$\gamma_T(\overline{\theta}) \int_T^\infty \frac{\lambda}{2} e^{-(\lambda+r)t-T}(\pi_m - \underline{\theta} - \sigma_t)dt + \gamma_T(\widetilde{\theta})(\pi_m - \underline{\theta})\frac{\lambda}{2(2\lambda+r)}[1 + \frac{\lambda}{2(\lambda+r)}] \geq \gamma_T(\widetilde{\theta}) \int_T^\infty \frac{\lambda}{2} e^{-(\lambda+r)(t-T)}\sigma_t dt, \tag{$IC(\widetilde{\theta} \to \overline{\theta})$}$$

$$r(\pi_m - \underline{\theta} - \sigma_T) \geq \frac{\lambda}{2}(2\sigma_T - \pi_m - \underline{\theta}) \tag{$IC(\underline{\theta} \to \widetilde{\theta})$}$$

$$\gamma_T(\overline{\theta}) \int_T^\infty \frac{\lambda}{2} e^{-(\lambda+r)t-T}(\pi_m - \underline{\theta} - \sigma_t)dt + \gamma_T(\widetilde{\theta})(\pi_m - \underline{\theta})\frac{\lambda}{2(2\lambda+r)}[1 + \frac{\lambda}{2(\lambda+r)}] \geq \pi_m - \widetilde{\theta} - \sigma_T, \tag{$IC(\widetilde{\theta} \to \underline{\theta})$}$$

Using these inequalities, we characterize conditions under which the efficient outcome can be achieved without subsidies:

Proposition 3 *The efficient outcome can be supported by the compensating mechanism without subsidies if and only if* $\frac{\lambda(\pi_m - \underline{\theta})}{2(2\lambda+r)}[1 + \frac{\lambda}{2(\lambda+r)}] \geq \pi_m - \widetilde{\theta}$.

[4] The details of the computation can be found in the appendix

Proposition 3 shows that it is possible to implement the first best outcome without any subsidies, i.e., without any payments to the inactive firm, when V_{EL}, the expected payoff obtained in the collusive benchmark when the firms wait to learn that the cost is low, exceeds $2(\pi_m - \widetilde{\theta})$, twice the expected payoff of a monopolist firm which invests immediately. This result stands in sharp contrast to the results obtained in the preceding subsection, when firms can only choose compensating payments *ex post*, after one of the firms has invested. The mechanism designer's ability to assign investment decisions allows him to reach the first best outcome, even without compensating payments, by using the fact that a firm which has not learned its cost may be unwilling to imitate a low cost firm and to invest. If the condition $V_{EL} > 2(\pi_m - \widetilde{\theta})$ is *not* satisfied, the implementation of the first best requires the use of subsidies.

5 Conclusion

This paper analyzes a model of entry with learning. Two firms contemplate entry into a new market, or the development of a new product and gradually learn about their private entry costs. We show that when signals are public, the model either results in a preemption game or a waiting game, and when signals are private, firms which have not learned their cost yet may choose to enter at a finite time, resulting in the same rent equalization phenomenon as in Fudenberg and Tirole (1985). As opposed to Hopenhayn and Squintani (2011), we find that preemption is greater when signals are private, because firms do not know whether the other firm has given up on entering the market. As compared to the collusive outcome, the equilibrium of the entry timing game exhibits three sources of inefficiencies: dissipation of the monopoly rent, duplication of entry costs and excess momentum.

We analyze how collusive schemes by which one firm pays the other firm to prevent it from entering the market can be implemented. We observe that, with compensating payments alone, collusion can only be effective if the first firm enters sufficiently early, and that these payments must allocate a significant share of the surplus to the excluded firm. We further analyze how such payments can be implemented if a third party, such as a venture capitalist or a granting agency, makes the joint investment decision. Here, we observe that collusion can be effective at any point in time without payments to the inactive firm as long as the expected benefits of learning the cost of entry are sufficiently high. If they are not sufficiently large, then the implementation of the collusive outcome requires again a sharing of the surplus between the active and the inactive firm.

We have framed our study as a market entry game. However, our model also covers two-stage R&D games, where firms first experiment to learn their cost in the research project, and then enter into a stochastic innovation race.

Our analysis belongs to an emerging literature on innovation races and timing games with private signals. It leaves a number of questions unanswered. What happens if signals are not perfect, and what is the effect of changes in the precision of the signals on preemption? What happens when uncertainty pertains to the common value of the innovation rather than the private value entry cost? What if firms can control the acquisition of information by choosing their level of (costly) experimentation? What happens if the two firms, rather than being independent agents, are two teams in an organization contracting with a principal? We plan to tackle these problems in future research.

References

[1] Akcigit, U. and Liu, Q., 2013. The Role of Information in Innovation and Competition. Mimeo, Department of Economics, University of Pennsylvania.

[2] Athey, S. and Segal, I., 2013. An Efficient Dynamic Mechanism. *Econometrica* (forthcoming).

[3] Chamley, C. and Gale, D., 1994. Information Revelation and Strategic Delay in a Model of Investment. *Econometrica* 62(5), 1065–1085.

[4] Choi, J.P., 1991. Dynamic R&D Competition under 'Hazard Rate' Uncertainty. *RAND Journal of Economics* 22, 596–610.

[5] Decamps, J.-P. and Mariotti, T., 2004. Investment timing and learning externalities. *Journal of Economic Theory* 118, 80–102.

[6] Fudenberg, D., and Tirole, J., 1985. Preemption and Rent Equalization in the Adoption of New Technology. *Review of Economic Studies* 52(3), 383–401.

[7] Gandal, N., and Scotchmer, S., 1993. Coordinating Research through Research Joint Ventures. *Journal of Public Economics* 51(2), 173–193.

[8] Goldfain, E., and Kovac, E., 2007. Financing of Competing Projects with Venture Capital. Mimeo, Department of Economics, University of Bonn.

[9] Gordon, S., 2011. Announcing Intermediate Breakthroughs in an R & D Race. Mimeo, Department of Economics, University of Montreal.

[10] Grossman, G.M. and Shapiro, C., 1987. Dynamic R&D Competition. *Economic Journal* 97, 372–387.

[11] Harris, C. and Vickers, J., 1985. Perfect Equilibrium in a Model of a Race. *Review of Economic Studies* 52, 193–209.

[12] Harris, C. and Vickers, J., 1987. Racing with Uncertainty. *Review of Economic Studies* 54, 1–21.

[13] Hopenhayn, H. and Squintani, F., 2011. Preemption Games with Private Information. *Review of Economic Studies* 78, 667-692.

[14] Hoppe, H.C. and Lehmann Grube, U., 2005. Innovation Timing Games: A General Framework with Applications. *Journal of Economic Theory* 121, 30–50.

[15] Katz, M. and Shapiro, C., 1987. R&D Rivalry with Licensing or Imitation. *American Economic Review* 77, 402–420.

[16] Keller, G., Cripps, M.W., and Rady, S., 2005. Strategic Experimentation with Exponential Bandits. *Econometrica* 73, 39–68.

[17] Keller, G. and Rady, S., 1999. Optimal Experimentation in a Changing Environment. *Review of Economic Studies* 66, 475–507.

[18] Laffont, J.J. and Martimort, D., 1998. Collusion under Asymmetric Information. *Econometrica* 65, 875–911.

[19] Lambrecht, B. and Perraudin, W., 2003. Real options and preemption under incomplete information. *Journal of Economic Dynamics & Control* 27, 619–643.

[20] Mason, R., and Weeds, H., 2010. Investment, Uncertainty and Preemption. *International Journal of Industrial Organization* 28, 278–287.

[21] Moldovanu, B. and Sela, A., 2001. The Optimal Allocation of Prizes in Contests. *American Economic Review* 91, 542–558.

[22] Moscarini G. and Squintani, F., 2010. Competitive Experimentation with Private Information: The Survivor's Curse. *Journal of Economic Theory* 145, 639–660.

[23] Murto, P. and Valimaki, J., 2011. Learning and Information Aggregation in an Exit Game. *Review of Economic Studies* 78, 1426–1461.

[24] Reinganum, J.F., 1982. A Dynamic Game of R and D: Patent Protection and Competitive Behavior. *Econometrica* 50, 671–688.

[25] Rosenberg, D., Solan, E., and Vieille, N., 2007. Social Learning in One Armed Bandit Problems. *Econometrica* 75, 1591–1611.

[26] Spatt, C. and Sterbenz, F., 1985. Learning, Preemption and the Degree of Rivalry. *RAND Journal of Economics* 16, 85–92.

[27] Weeds, H., 2002. Delay in a Real Options Model of R&D Competition. *Review of Economic Studies* 69(3), 729–747.

A Proofs

A.1 Proof of Lemma 1

Clearly, if the other firm has already entered, it is a dominant strategy to enter immediately as $\pi_d > \underline{\theta}$. Suppose that the other firm has not entered yet, and consider a firm which learns that its cost is low at date T. By delaying investment to date $T + \Delta$, the firm loses the positive profit made between T and $T + \Delta$, equal to $(1 - d)\pi_d$ or $(1 - \delta)\pi_m$, depending on whether the other firm invests at period T or not. In addition, let $\Delta g(T)$ be the probability that a firm which did not learn its cost invests between T and $T + \Delta$. By investing at date T a low cost firm will block the entry of the competitor which has not yet learned its cost, resulting in an expected payoff of

$$\sum_{t=\frac{T}{\Delta}+1}^{\infty} (1-\lambda\Delta)^{t-1-\frac{T}{\Delta}} \frac{\lambda\Delta}{2}(\pi_m - \underline{\theta}) + \sum_{t=\frac{T}{\Delta}+1}^{\infty} (1-\lambda\Delta)^{t-1-\frac{T}{\Delta}} \frac{\lambda\Delta}{2}(\pi_m(1-\delta^{t-\frac{T}{\Delta}}) + \pi_d\delta^{t-\frac{T}{\Delta}} - \underline{\theta})$$

By contrast, if the firm waits until $T+\Delta$, and the competitor invests, it will get an expected payoff of $\pi_d - \underline{\theta}$. Hence, by delaying investment, the firm loses an expected payoff of

$$\Delta g(T)(\pi_m - \pi_d)[1 - \sum_{t=\frac{T}{\Delta}+1}^{\infty} (1-\lambda\Delta)^{t-1-\frac{T}{\Delta}} \frac{\lambda\Delta}{2} \delta^{t-\frac{T}{\Delta}}].$$

A.2 Proof of Proposition 1

We focus attention on the optimal strategy of a firm when the other firm has not received any signal yet. For $\Delta \to 0$, the probability that the other firm has received a signal between period $t-1$ and period t goes to 0. In that case, the investment game played by the two firms is given by the following bi-matrix game, where $W(t+1)$ denotes the continuation value for both players of entering into period $t+1$.

	invest	not invest
invest	$(\pi_d - \widetilde{\theta}, \pi_d - \widetilde{\theta})$	$(V_L - \widetilde{\theta}, V_F)$
not invest	$(V_F, V_L - \widetilde{\theta})$	$(W(t+\Delta), W(t+\Delta))$

Table 5: Investment game played by the two firms is none of them has invested up to date t and costs are not known.

We first consider a symmetric equilibrium where both firms invest with positive probability $p \in (0,1)$. In that equilibrium,

$$W(t) = p(\pi_d) + (1-p)V_L - \widetilde{\theta},$$

and

$$W(t) = pV_F + (1-p)W(t+1).$$

Solving this equation and letting the time between two successive periods go to 0, we find

$$p = \frac{V_L - \widetilde{\theta} - V_F}{V_L - \pi_d},$$

showing that an equilibrium with preemption exists if and only if $V_L - V_F = \pi_m - V_{E_1} \geq \widetilde{\theta}$.

Next, suppose that $V_L - V_F = \pi_m - V_{E_1} < \widetilde{\theta}$. We construct a symmetric equilibrium where both firms choose to delay investment. By delaying investment by one period when the other firm does not invest, a firm obtains an expected payoff of:

$$\begin{aligned} W(t+1) &= \exp -r\Delta[(\frac{\lambda\Delta}{2})^2(\pi_d - \underline{\theta}) + \frac{\lambda\Delta}{2}(1-\lambda\Delta)(V_L - \underline{\theta}) + (\frac{\lambda\Delta}{2})^2(\pi_m - \underline{\theta}) \\ &+ \frac{\lambda\Delta}{2}(1-\lambda\Delta)V_F + \frac{\lambda\Delta}{2}(1-\lambda\Delta)V_{E_1} + (1-\lambda\Delta)^2(V_L - \widetilde{\theta})] \end{aligned}$$

Letting $\Delta \to 0$,

$$W(t+1) = V_L - \widetilde{\theta} + \frac{\lambda\Delta}{2}(V_L - V_F - \underline{\theta} + V_{E_1}) - (r+2\lambda)\Delta(V_L - \widetilde{\theta}) + \mathcal{O}(\Delta^2).$$

Now compute

$$\begin{aligned} V_L - V_F - \underline{\theta} + V_{E_1} &= \pi_m - \underline{\theta} \\ &> 2\pi_d - \underline{\theta} \\ &> 2(\pi_d - \underline{\theta}) \end{aligned}$$

and, as $V_F > V_L - \widetilde{\theta}$,

$$\begin{aligned} V_L - \widetilde{\theta} &< V_F \\ &< \frac{\lambda}{2(\lambda+r)}(\pi_d - \underline{\theta}), \end{aligned}$$

establishing that, as $\Delta \to 0$, $\frac{\lambda\Delta}{2}(V_L - V_F - \underline{\theta} + V_{E_1}) - (r+2\lambda)\Delta(V_L - \widetilde{\theta}) < 0$ and hence $W(t+\Delta) > V_L - \widetilde{\theta}$, so that firms always have an incentive to wait.

A.3 Proof of Theorem 1

As in the proof of Proposition 1, we first consider a symmetric equilibrium where both firms invest with positive probability $p(T) \in [0,1]$ at time T. Taking $\Delta \to 0$, the probability that the rival firm has received a low cost signal between $T - \Delta$ and T goes to 0, and we compute the expected value of a firm which invests as:

$$U(1) = \gamma_T(\widetilde{\theta})p(T)(\pi_d - \widetilde{\theta}) + \gamma_T(\widetilde{\theta})(1-p(T))(V_L - \widetilde{\theta}) + \gamma_T(\overline{\theta})(\pi_m - \widetilde{\theta}),$$

and the expected value of a firm which does not invest as

$$U(0) = \gamma_T(\widetilde{\theta})p(T)V_F + (\gamma_T(\widetilde{\theta})(1 - p(T)) + \gamma_T(\overline{\theta}))W(T + \Delta).$$

In a symmetric equilibrium with positive exit probabilities, $W(T) = U(0) = U(1)$ and as $\Delta \to 0$, $W(T + \Delta) \to W(T)$, resulting in

$$p(T) = \frac{V_L(T) - V_F - \widetilde{\theta}}{\gamma_T(\widetilde{\theta})(V_L - \pi_d)}.$$

so that, this equilibrium exists if and only if $V_L(T) \geq V_F - \widetilde{\theta}$. This shows that in case (i) preemption arises at time zero, and in case (ii), firms rush to invest at time $\tilde{T}$. We now consider a situation where $V_L(T) < V_F - \widetilde{\theta}$ and show that firms have an incentive to wait. Suppose that the other firm does not invest and compute the expected utility of waiting one period before investing:

$$\begin{aligned} W(T + \Delta) &= \exp -r\Delta[(\frac{\lambda\Delta}{2})^2(\pi_d - \underline{\theta}) + \frac{\lambda\Delta}{2}[\gamma_{T+\Delta}(\widetilde{\theta})(V_L - \underline{\theta}) + \gamma_{T+\Delta}(\overline{\theta})(\pi_m - \underline{\theta}) \\ &+ \frac{\lambda\Delta}{2}(1 - \lambda\Delta)V_F + (1 - \frac{\lambda\Delta}{2})(1 - \lambda\Delta)(V_L(T + \Delta) - \widetilde{\theta})] + \mathcal{O}(\Delta^2) \end{aligned}$$

As $\Delta \to 0$,

$$W(T + \Delta) = \frac{\lambda}{2}(V_F + \widetilde{\theta} - \underline{\theta}) - (r + \lambda)(V_L(T) - \widetilde{\theta}) + (V_L(T) - \widetilde{\theta}).$$

Next,

$$\begin{aligned} \frac{2(r + \lambda)}{\lambda}(V_L(T) - \widetilde{\theta}) &\leq \frac{2(r + \lambda)}{\lambda}V_F, \\ &= (\pi_d - \underline{\theta}), \\ &< V_F + \widetilde{\theta} - \underline{\theta}, \end{aligned}$$

so that $W(T + \Delta) > V_L(T) - \widetilde{\theta}$, implying that the firm has an incentive to wait.

A.4 Proof of Proposition 2

In order to implement the cooperative benchmark, two conditions must be satisfied: (i) no firm must be willing to enter the market at $t < t^*$ if it has not learned its cost and (ii) a firm which learns that it has a low cost must be willing to enter the market immediately. The first condition will hold as long as:

$$U_F(t) > U_L(t) - \widetilde{\theta}.$$

As $U_L(t) = \pi_m - U_F(t)$, this results in

$$2U_F(t) > \pi_m - \widetilde{\theta}.$$

For the second condition to hold, we characterize the conditions under which an equilibrium where a firm immediately invests after it observes that its cost is low exists. The discounted expected payoff of investing at period t when the other firm does not invest is:

$$W(t) = U_L(t) - \underline{\theta},$$

whereas by waiting one period the firm will obtain a discounted expected payoff of

$$W(t+\Delta) = e^{-r\Delta}[(1 - \frac{e^{-\mu\Delta}}{2})U_F(t+\Delta) + \frac{e^{-\mu\Delta}}{2}U_L(t+\Delta)].$$

For Δ small enough and assuming that utilities are differentiable,

$$W(t+\Delta) - W(t) = \Delta[(-2r-\mu)U_L(t) + \mu U_F(t) + U_L'(t)]$$

so that the firm has an incentive to enter immediately if and only if:

$$2U_F(t) < \frac{2r+\mu}{r+\mu}\pi_m + \frac{U_F'(t)}{r+\mu}.$$

A.5 Derivation of expected payoffs when $\Delta \to 0$

Recall that, as Δ goes to 0,

$$(1-\lambda\Delta)^{\frac{1}{\Delta}} \to e^{-\lambda},$$

and that the instantaneous probability that the signal is received between two periods $t\Delta$ and $(t+1)\Delta$ converges to λ. Hence, we compute the expected payoff of a firm with high cost as

$$EV_T(\overline{\theta}) = \gamma_T(\widetilde{\theta}) \int_T^\infty \frac{\lambda}{2} e^{-(\lambda+r)(t-T)}\sigma_t dt.$$

Similarly,

$$
\begin{aligned}
EV_T(\widetilde{\theta}) &= \gamma_T(\overline{\theta}) \int_T^\infty \frac{\lambda}{2} e^{-(\lambda+r)(t-T)} (\pi_m - \underline{\theta} - \sigma_t) dt \\
&+ \gamma_T(\widetilde{\theta}) \int_T^\infty \frac{\lambda}{2} e^{-(2\lambda+r)(t-T)} [\pi_m - \underline{\theta} + \int_t^\infty \frac{\lambda}{2} e^{-(\lambda+r)(\tau-t)} (\pi_m - \underline{\theta}) d\tau] dt, \\
&= \gamma_T(\overline{\theta}) \int_T^\infty \frac{\lambda}{2} e^{-(\lambda+r)(t-T)} (\pi_m - \underline{\theta} - \sigma_t) dt + \gamma_T(\widetilde{\theta})(\pi_m - \underline{\theta}) \frac{\lambda}{2(2\lambda+r)} [1 + \frac{\lambda}{2(\lambda+r)}].
\end{aligned}
$$

Finally, we compute $IC(\underline{\theta} \to \widetilde{\theta})$ as $\Delta \to 0$ using the fact that $\sigma_{T+1} \to \sigma_T$ and $\delta = e^{-r\Delta} \equiv 1 - r\Delta$,

$$
\begin{aligned}
\frac{\lambda\Delta}{2} \frac{\pi_m - \underline{\theta}}{2} + (1 - \frac{\lambda\Delta}{2} (\pi_m - \underline{\theta} - \sigma_T) &\geq \frac{\lambda\Delta}{2} \sigma_T \\
&+ (1 - \frac{\lambda\Delta}{2})(1 - r\Delta)[\frac{\lambda\Delta}{2} \frac{\pi_m - \underline{\theta}}{2} + (1 - \frac{\lambda\Delta}{2})(\pi_m - \underline{\theta} - \sigma_T)].
\end{aligned}
$$

Focusing on the terms in the first order in Δ, this inequality becomes

$$
\begin{aligned}
\frac{\lambda}{2}(\frac{\pi_m - \underline{\theta}}{2} - \sigma_T) - \frac{\lambda}{2}(\pi_m - \underline{\theta} - \sigma_T) - \frac{\lambda}{2} \frac{\pi_m - \underline{\theta}}{2} \\
+ r(\pi_m - \underline{\theta} - \sigma_T) + \frac{\lambda}{2}(\pi_m - \underline{\theta} - \sigma_T) &\geq 0,
\end{aligned}
$$

resulting in

$$
r(\pi_m - \underline{\theta} - \sigma_T) \geq \frac{\lambda}{2}(2\sigma_T - \pi_m - \underline{\theta}).
$$

A.6 Proof of Proposition 3

When $\sigma_T = 0$ for all T, the only binding constraint is the constraint $IC(\widetilde{\theta} \to \underline{\theta})$ which reads:

$$
\gamma_T(\overline{\theta})(\pi_m - \underline{\theta}) \frac{\lambda}{2(\lambda+r)} + \gamma_T(\widetilde{\theta})(\pi_m - \underline{\theta}) \frac{\lambda}{2(2\lambda+r)} [1 + \frac{\lambda}{2(\lambda+r)}] \quad \geq \quad \pi_m - \widetilde{\theta}.
$$

As

$$
\frac{\lambda}{2(\lambda+r)} > \frac{\lambda}{2(2\lambda+r)} [1 + \frac{\lambda}{2(\lambda+r)}],
$$

this condition is hardest to satisfy at $T = 0$ when $\gamma_T(\widetilde{\theta}) = 1$, so that the incentive condition is satisfied for all T if and only if

$$
\frac{\lambda(\pi_m - \underline{\theta})}{2(2\lambda+r)} [1 + \frac{\lambda}{2(\lambda+r)}] \geq \pi_m - \widetilde{\theta}.
$$