Deriving the Global-Game Selection in Games with many Actions and Asymmetric Players

Frank Heinemann*

October 2015

Preliminary and incomplete version

Abstract:

This paper first shows how to apply the global-game selection to a class of models with multiple Nash equilibria due to strong strategic complementarities. Because of the technical complications in deriving the equilibrium, global games have (so far) only been applied to symmetric binary action games. Based on new results for the equilibrium characterization, this paper shows how to decompose games with many actions or different player types in such a way that simple solution techniques can be applied for deriving the global-game selection. The paper explains the solution technique, provides examples, and demonstrates (by comparing solutions with experimental evidence) how global games can be used as a descriptive or normative theory.

The second (and yet not elaborated) part of this paper discusses the predictive power of the global game selection in experiments, focussing on a coordination game with asymmetric players.

JEL codes: C72, D82

Keywords: coordination games, equilibrium selection, global game, private information

1. Introduction

The theory of global games has been introduced by Carlsson and van Damme (1993) as a new solution concept for coordination games with multiple equilibria. The global-game approach relaxes the assumption that the game is common knowledge among players. It embeds the game to be analyzed in a larger class of games. The particular game is then assumed to be randomly drawn out of this world of possible games (which explains the term *global* game). Players are not perfectly informed about the selected game, but receive private signals, instead. They are, however, perfectly rational in analyzing their information and deducing the strategies of other players in the global game. The class of games and the distribution of

* Berlin University of Technology (TU Berlin), e-mail address frank.heinemann@tu-berlin.de

signals are common knowledge, so that standard equilibrium concepts can be applied to the global game.

Assuming a particular embedding (class of games) and a particular functional form of the signals' distribution allows estimating one parameter for the variance of private signals for a given set of data from actual players in an experiment (see Heinemann, Nagel, and Ockenfels (2009) for an example). Such an estimated global game may serve as descriptive theory: asssuming external validity, the estimated parameter can be used to predict a probability distribution on the set of strategies for related games. In the limit, when the variance of private signals shrinks to zero, the global game selects a unique equilibrium of the original game that has multiple equilibria under common knowledge. This limit equilibrium is called *global-game selection*.[1] It is, thus, sufficient to assume a tiny bit of uncertainty for getting a unique equilibrium. Since in reality common knowledge is extremely hard to establish anyway, the global-game approach has convinced many theorists of providing a reasonable selection mechanism. While it is highly unlikely that any set of real players will all choose the same equilibrium strategy from the start (thus, the limit is not descriptive for one-shot games), the selected strategy combination may well serve as a descriptive theory for repeated games, as positive theory for one-shot games, or as a benchmark to exploit the comparative statics properties of the selected equilibrium for further (applied) results.

Applications cover a wide range of topics: currency and banking crises, government debt and twin crises, refinancing of short-term credit to firms, competition between trading venues, decisions to join a revolution, poverty trap models, marketing of network goods, and antitrust regulation. All applications so far are binary-choice games and most papers use symmetric games. While some papers explicitly discuss the effects of changes in the probability distribution, in particular for analyzing the effects of balance sheet transparency, many papers concentrate on the global game selection as a benchmark that allows analyzing the equilibrium properties algebraically, including comparative statics.

A global game extends a supermodular game by defining an extended payoff function u that depends on a state variable θ and coincides with the payoff function of the original game at some realization of the state variable θ^* that may be normalized to zero. Furthermore, the global game introduces a probability distribution ϕ on the state space and a joint probability distribution f for players' private signals conditional on the realized state. Assuming that f has a bounded support, Frankel, Morris, and Pauzner (2003) have shown that the limit equilibrium for the variance of private signals shrinking to zero selects a unique strategy for almost any realization of the state variable. Consequently, this limit equilibrium selects a unique strategy for almost any supermodular game. This selection is called global-game selection (GGS). However, the GGS may depend on the way that the original game is

[1] The term global-game selection has been introduced by Heinemann, Nagel and Ockenfels (2009). Uniqueness holds for almost any game with strategic complementarities (details are provided in the formal section).

embedded. Thus, the multiplicity of equilibria may just be replaced by multiple selections depending on the global game that can be arbitrarily constructed. In order to avoid replacing one multiplicity by another, it needs to be shown that the GGS is independent of the three elements constituting a global game (u, ϕ, f). Frankel, Morris and Pauzner (2003) show indeed that the GGS is independent of ϕ, but may depend on f. They and Morris and Shin (2003) discuss examples in which the GGS does depend on f and sufficient conditions for the GGS being independent from f. This independence is called noise independence. In particular, it is known that games with a sufficiently small number of players and strategies are noise independent.

In particular, Frankel, Morris, and Pauzner (2003) show that symmetric binary choice games are noise independent. Morris and Shin (2003) show that the GGS of these games is the best response of a player who believes that the proportion of other players choosing either action has a uniform distribution in [0,1]. Thus, in symmetric binary choice games the GGS can be easily derived and analyzed algebraically. This is also the main reason, why many application concentrate on this class of models.

Basteck, Daniëls, and Heinemann (2013), henceforth BDH have shown that the GGS is independent of u and they complete the characterization of games for which noise independence can be established by counting the number of players and strategies. They also provide a decomposition result, establishing that the global game selection of a large supermodular game can be established by decomposing the game into smaller adjacent games. If the global game selections of these smaller games all point into the direction of the same strategy combination, then this combination is the GGS of the entire game. Furthermore, noise independence is inherited from the smaller games.

The aim of this paper is twofold: first, it shows how the decomposition result can be applied to games with more than two actions or with asymmetric players. Secondly, it confronts the theoretical results with experimental evidence and draws conclusions for the validity of global-game models and the GS in particular, as positive and normative theories.

By defining an appropriate decomposition of the set of strategy combinations, many at first sight complicated games can be decomposed into symmetric binary action games for which the GGS is almost trivial. This procedure avoids defining an explicit global game, analyzing its equilibria for a positive variance of signals, and explicitly taking the equilibrium conditions to the limit for vanishing noise. Instead, the decomposed games can be solved by the heuristic criterion from Morris and Shin (2003), explained above, so that the GGS of the large game can be described further analyzed algebraically.

The paper will concentrate on three examples: one is a minimum effort game that has been used by Bryant (1983) for describing the production of an intermediate good. In its abstract form, the game has been tested experimentally by Van Huyck, Battalio, and Beil (1990) and various other experimental economists in variations. The second example exploits the solution technique for showing that refinancing crises (bank runs) can be avoided by providing a third alternative between extending and withdrawing credit. The third alternative can be thought of being collateralized debt. The third option is constructed in such a way that the GGS of the game with three alternatives is "extend credit," while the GGS of the game without this third option is "withdraw credit." In this limit equilibrium, no player actually demands collateralized debt. The mere presence of the third alternative turns the selection around from the inefficient to the efficient equilibrium of the refinancing game. The third example is motivated by the literature on introducing network goods. The benefits from using a certain network good differ between individuals making the decision to buy a network good a binary-choice game with asymmetric players. The set of strategy combinations, however, can be decomposed in a way that allows applying the decomposition result, such that the smaller games consist are symmetric binary action games for which a simple solution technique applies. It is then easy to see, under which conditions the decomposition yields a unique and noise independent GGS.

The first and the third example have been tested in laboratory experiments. In the last part, this paper compares the theoretical predictions obtained from the GGS with empirical results from experiments and discusses whether and how the GGS can be used as descriptive or normative theory.

2. Definition of a global game and global-game selection

Let us start by introducing some notation borrowed from BDH. We denote the set of players by I. Each player i has an ordered finite action set $A_i = \{0,1,2, \ldots ,m_i\}$. Actions are denoted by $a_i \in A_i$, an action profile by $a \in A = \prod_{i \in I} A_i$. The lowest and highest action profiles are then given by 0 and m.

A complete information game Γ is specified by payoff functions $g_i : A \rightarrow \mathbf{R}$. Game Γ is **supermodular** [actions are strategic complements], if for all i and for all and for all $a_i \leq a'_i$ and $a_{-i} \leq a'_{-i}$:

$$g_i(a'_i, a_{-i}) - g_i(a_i, a_{-i}) \leq g_i(a'_i, a'_{-i}) - g_i(a_i, a'_{-i}).$$

Supermodularity implies that best response functions are non-decreasing. Supermodular games often have multiple equilibria. That case has been phrased as a case of strong strategic complementarities by Angeletos and Pavan (2004).

Following Frankel, Morris, and Pauzner (2003), a **global game** $G^v(u,\phi,f)$ is defined by

- payoff functions $u_i(a_i, a_{-i}, \theta)$, where $\theta \in \mathbf{R}$ is called state parameter, such that

(A1) for each θ, the complete information game, given by $u_i(\cdot,\theta)$ is a supermodular game, (A2) $\exists \underline{\theta}$ and $\overline{\theta}$, such that the lowest and highest action are strictly dominant in the games given by $u_i(\cdot,\underline{\theta})$ and $u_i(\cdot,\overline{\theta})$,

(A3) each u_i satisfies weak state monotonicity, which means that for all i and $a_i < \tilde{a}_i$, the payoff difference $u_i(\tilde{a}_i, a_{-i}, \theta) - u_i(a_i, a_{-i}, \theta)$ is weakly increasing in θ. This implies that higher states make higher actions more appealing.

- a distribution for the state parameter with continuous density ϕ, and
- a tuple of density functions f_i with finite support and a scale parameter $\nu \in (0,1]$. In the global game, players do not observe state θ. Instead, each player i receives a private signal $x_i = \theta + \nu \eta_i$, where the idiosyncratic noise term η_i is distributed according to density function f_i.

A global game G embeds a complete information game Γ at state θ^*, if $g_i(a) = u_i(a, \theta^*)$ for all players i and for all action profiles a.

Theorem (analogue to Frankel, Morris, and Pauzner (2003):
As the scale parameter v goes to zero, the global game $G^v(u,\phi,f)$ has an essentially unique limit equilibrium.

More precisely, denote a pure strategy of the global game by $s_i: \; R \to A_i$, such that player i chooses action $s_i(x_i)$ when receiving signal x_i. There is a strategy combination s, such that for $v \to 0$, any equilibrium $s^v(x)$ of $G^v(\cdot)$ converges to $s(x)$ for all x except possibly at the finitely many discontinuities of s.

If the global game‘s limit equilibrium strategy profile is continuous at state θ^*, its value at that state determines a particular Nash equilibrium of the complete information game, called **global-game selection** (GGS).

The first question that arises, when defining a selection for a class of games with multiple equilibria, is, whether the selection is actually unique. Since a complete information game can be extended to a many different global games, distinguished by the extended payoff function u, the prior distribution of the state variable ϕ, and a tupel of noise distributions for private signals f, we would like to know under which conditions is the GGS independent from u, ϕ, and f ? If it is not independent, then multiple Nash equilibria of the underlying complete information game are replaced by potentially different limit equilibria of the different global games. Frankel, Morris, and Pauzner (2003) show that the GGS is independent from ϕ. BDH show that the GGS is independent from u. The combination of these two results implies that one may use without loss of generality a particular global-game embedding, such as $u_i(a,\theta) = g_i(a) + \theta a_i$ (BDH).

Proof: For a sufficiently wide support of ϕ, u_i satisfies the global-game assumptions (A1) to (A3). Obviously, u_i embeds g at θ^*=0.

Unfortunately, the GGS may depend on *f*. This is known since Frankel, Morris, and Pauzner (2003) and Morris and Shin (2003) constructed the first examples of global games in which the limit equilibria for $v \to 0$ depend on the distribution *f*. The GGS is called **noise independent**, if the GGS is independent of the particular density function of private signals *f*. On the other hand, Carlsson and Van Damme (1993) had already shown that for any two-player-two-action game, the GGS is independent of *f*. In symmetric 2-player-2-action games, the GGS is actually identical to the risk-dominant equilibrium defined by Harsanyi and Selten (1988). Table 1, taken from BDH gives an overview of the games, for which noise independence can be established simply by counting the number of players and actions. It shows that symmetric complete-information games with two actions for each player, symmetric 2-player games with 3 actions for each player and asymmetric 2-player games, I which at least one of the players can only choose between two possible actions are noise independent. In these games, the GGS can be calculated by solving the simplest possible global-game. Larger games, however, may not be noise independent. For these games, noise independence can be established by using *p*-dominance or potential maximizers, arguably complicated concepts that most applied researchers do not want to go into.

Symmetric Games			
actions:	2 each	3 each	4 each
2 players	✓[a]	✓[c]	×[b]
3 players	✓[b]	×[d]	
n players	✓[b]		

Asymmetric Games			
actions:	2 each	2 by n	3 each
2 players	✓[a]	✔[g]	×[c]
3 players	×[e]	n/a	
n players	×[f]	n/a	

✓ Always noise independent. × Counterexample to noise independence exists. [a]Carlsson and Van Damme [6]. [b]Frankel, Morris and Pauzner [10]. [c]Basteck and Daniëls [1]. [d]Basteck et al. [2]. [e]Carlsson [4]. [f]Corsetti et al. [7]. [g]This paper, see Section 5: Two-player games with 2-by-n-actions. For empty cells noise dependence follows from an example in smaller games.

TABLE 1. Noise (In)dependence in Supermodular Games

It is therefore quite helpful that BDHH show that some larger games can be broken down into small games, for which noise independence can be established by counting the number of players and actions. The idea behind their theorem rests on the observation that any equilibrium of a global game is step function with equilibrium strategy profiles increasing in the relevant state. Hence, the GGS is also increasing in the state that embeds the complete information game. When the variance of idiosyncratic noise terms approaches zero, players may only need to consider two action profiles, namely those that are played for somewhat smaller and somewhat larger states of the world. This rough intuition can not always be successful in describing a GGS, because we know that 2-action games are noise independent and because we know examples of larger games that are not. However, with a simple criterion for games that can be broken down, may reduce the workload for applied researchers to a minimum.

To get this, consider a supermodular complete information game Γ with joint action set A. For action profiles $a \leq a'$, we define the set of action profiles between these two:

$$[a,a'] = \{\tilde{a} \in A \mid a \leq \tilde{a} \leq a'\}.$$

Now, we look at the restricted game $\Gamma|[a,a']$, which is given by restricting the joint action set of Γ to the action profiles a and a' (inclusive). BDH prove the following Lemma:

Lemma (BDH, 2013): Consider a supermodular game Γ and a noise structure f. An action profile a^n is the unique GGS of Γ, if there is a sequence $0 = a^0 \leq a^1 \leq \ldots \leq a^n \leq \ldots \leq a^m = m$ s.t.

(i) a^j is the unique GGS in $\Gamma|[a^{j-1}, a^j]$ for all $j \leq n$, and

(ii) a^{j-1} is the unique GGS in $\Gamma|[a^{j-1}, a^j]$ for all $j > n$.

Corollary: If all the restricted games are noise independent, then Γ is also noise independent and a^n is the unique noise independent GGS of Γ.

This result provides a simple solution technique: If you have a game with multiple equilibria, first check whether it is supermodular, so that the result applies. Then decompose the game by defining restricted sets of action profiles that (if patched together) stretch from the lowest action profile (denoted by 0) to the largest action profile m. Note that it is not necessary that all action profiles of the original game are contained in one of the restricted sets. We only need the highest action profile of one set being the lowest of the next set. So, you basically need to define a sequence of profiles $0 = a^0 \leq a^1 \leq \ldots \leq a^n \leq \ldots \leq a^m = m$.

Then, derive the GGS for each of the restricted games. This may sound cumbersome, but actually the trick is in defining an appropriate sequence of profiles such that the GGS can be calculated without going through the full blown global game. Now, if all solutions point to the same strategy profile, this profile a GGS of the large game. We can simply mark these selections by arrows as in the following example, where a^3 is the GGS of the large game:

$$0 = a^0 \rightarrow a^1 \rightarrow a^2 \rightarrow a^3 \leftarrow a^4 \leftarrow a^5 = m$$

If, in addition, all small games are noise independent, the large game is also noise independent. Since noise independence is guaranteed for symmetric 2-action games, it is advisable to define the sequence of strategy profiles in such a way, that all restricted games fall into this class.

This is certainly not possible for all games, and even if a large can be broken down into restricted 2-action games, the arrows may not always point into direction of the same strategy. In the following example, the arrows point to strategy profiles a^1 and a^4 and we cannot say which or whether any of these two profiles is the GGS of the large game:

$$0 = a^0 \rightarrow a^1 \leftarrow a^2 \rightarrow a^3 \rightarrow a^4 \leftarrow a^5 = m$$

In such cases, we cannot use the decomposition result and need other techniques for calculating the large game's GGS and for checking whether it is noise independent. However, the applications in the next section demonstrate that the decomposition may be quite helpful for a variety of games that could otherwise not easily be solved.

3. Applications

Application 1: Minimum effort game

The first application is a minimum effort game as it has been introduced by Bryant (1983) and Van Huyck et al (1990).

Each player *i* produces an intermediate good, necessary for a final good.

Players choose production levels $a_i \in \{0,...,m\}$.

Production of the final good is $a_{\min} = \min\{a_i \mid i \in I\}$.

Players' payoff functions are $g(a) = b(a_{\min}) - c(a_i)$,

where *b* and *c* are increasing benefit and cost functions.

The game is a supermodular with many actions and symmetric payoffs.

Decompose the game into *m* binary-action games with joint action sets $\{k-1, k\}, \quad 0 < k \le m$.

Van Huyck et al. (1990): $a_i \in \{1,\ldots,7\}, \quad g(a) = b\,a_{\min} - c\,a_i, \quad b > c.$

Binary-action games $[a, a+1]$

Symmetric *n*-player-2-action games are noise independent (FMP, 2003).

In symmetric 2-action games, the GGS is the best response to the belief that the proportion of other players choosing either action has a uniform distribution (Morris and Shin, 2003).

=> probability that all *n*-1 other players choose *a*+1 is 1/*n*.

=> expected payoff action *a* + 1 is

$$b\left[\frac{1}{n}(a+1) + \frac{n-1}{n}a\right] - c(a+1) = b/n + ba - c(a+1)$$

=> payoff from action *a* is $(b-c)\,a$

=> GGS selects *a* if $b/n < c$.

As we can see from this, it is very simple to establish the GGS for this game.[2] Furthermore, we know that the GGS is noise independent, because this property is inherited from the restricted games. This is also the reason, why we can apply a very heuristic solution technique: In symmetric 2-action games, the GGS is the best response to the belief that the

[2] Using robustness, BDH show that uniqueness and noise independence can also be established for a generalized version with asymmetric payoff functions. However, the general solution has no closed-form algebraic solution. Here, we concentrate on the symmetric game that has been used in experiments and allows applying the decomposition result of BDH.

proportion of other players choosing either action has a uniform distribution (Morris and Shin, 2003). Note that it is not necessary to define any noise distributions here, and then take the scale parameter to its limit of zero. The reason why we can circumvent this messy procedure of defining an explicit global game is that these games are noise independent.

Van Huyck et al. (1990) conducted an experiment on repeated minimum effort games. The parametrs they chose were $b=0.2$, $c=0.1$, and $n \in \{2,14,16\}$.

Their results were that groups of size $n \geq 14$, converged to lowest action, which is in line with the GGS. If $n=2$, the GGS does not yield a clear prediction. Here, they observed that a vast majority of groups converged to the efficient strategy combination, in which both subjects chose the highest action. It seems to be a robust finding that subjects in experiments deviate from the GGS towards strategies with higher efficiency. [quote some other papers on this]

Application 2: Refinancing game

This game is taken from BDH. Daniëls (2013) uses it for arguing that collateralized lending may lead to inefficient investment.

Consider a standard refinancing game with 3 lenders. Payoffs are given by the following payoff table:

		Decisions of other lenders		
		(0,0)	(0,2) or (2,0)	(2,2)
		Total default	Partial default	Success
	0 = withdraw	1	1	1
Lender *i*				
	2 = Roll over	0	2/3	2

From Frankel, Morris, and Pauzner (2003), we know that symmetric 3-player-2-action gamea are noise independent. Furthermore, in symmetric 2-action games, the GGS is the best response to the belief that the proportion of other players choosing either action has a uniform distribution (Morris and Shin, 2003). This allows us to derive the GGS easily:

=> expected payoff of rolling over = 2.667 / 3 < 1

=> GGS selects withdrawal.

The selected equilibrium is inefficient. It resembles a self-fulfilling financial crisis. Creditors withdraw their credit, because they believe that others withdraw. However, the firm or bank could succeed if all creditors would roll-over their credit and this would also be in the interest of creditors. Bank runs, speculative currency crises, and systemic liquidity crises are of this nature. It is therefore worthwhile thinking about how such a crisis can be avoided. Diamond and Dybvig (1983) discuss deposit insurance and suspension of convertibility as measures for avoiding bank runs. In the light of global games we can suggest another possible route: Suppose that we introduce a third option besides rolling over and withdrawal that we may call „collateralized debt". Collateralized debt reduces the potential losses for a lender if the bank fails. Hence the payoffs in cases of a total or partial default are higher for a lender who has chosen collateralized debt than for a lender with uncollateralized credit. This insurance, however, comes at a cost: in case the bank succeeds, the returns to the collateralized lender are smaller than to an unsecured one. The numerical payoffs for this example are given below.

		Decisions of other lenders		
		(0,0) total	(0,1), (1,1), or (0,2) partial default	(1,2) or (2,2) success
	0 = withdraw	1	1	1
Lender *i*	1 = collateralized debt	2/3	4/3	1.5
	2 = roll over	0	2/3	2

Collateralized debt gives lenders an additional payoff (2/3) in case case of total or partial default, but reduces payoff (by 0.5) if the project succeeds.

As we know from an example by Basteck et al. (2010), symmetric 3-player-3-action games may be noise dependent. Hence, calculating the GGS may require defining a particular global game with explicit noise distributions and characterizing the GGS by the equilibrium in the limit for vanishing noise. Instead, we decompose the game into two 3-player-2-action games with action sets {0,1} and {1,2}. Both are supermodular and noise independent.

Applying the heuristic criterion by Morris and Shin (2003), we can easily see that the expected payoff of a player who chooses collateralized debt and attaches equal probabilities to 0, 1, or 2 of the other players choosing collateralized debt (and the others withdrawing) is (2/3 + 4/3 + 4/3)/3 = 10/9 > 1. Hence, the GGS of game [0,1] is action 1.

Similarly, in the restricted game [1,2], the expected payoff from action 1 is (4/3+1.5+1.5)/3 = 4.33, and the expected payoff from action 2 is (2/3+2+2)/3=4.66. Hence, the GGS of game [1,2] is action 2.

In both restricted games, the respective higher action is selected. Thus, the noise idependent GGS of the 3x3-game is „roll over".

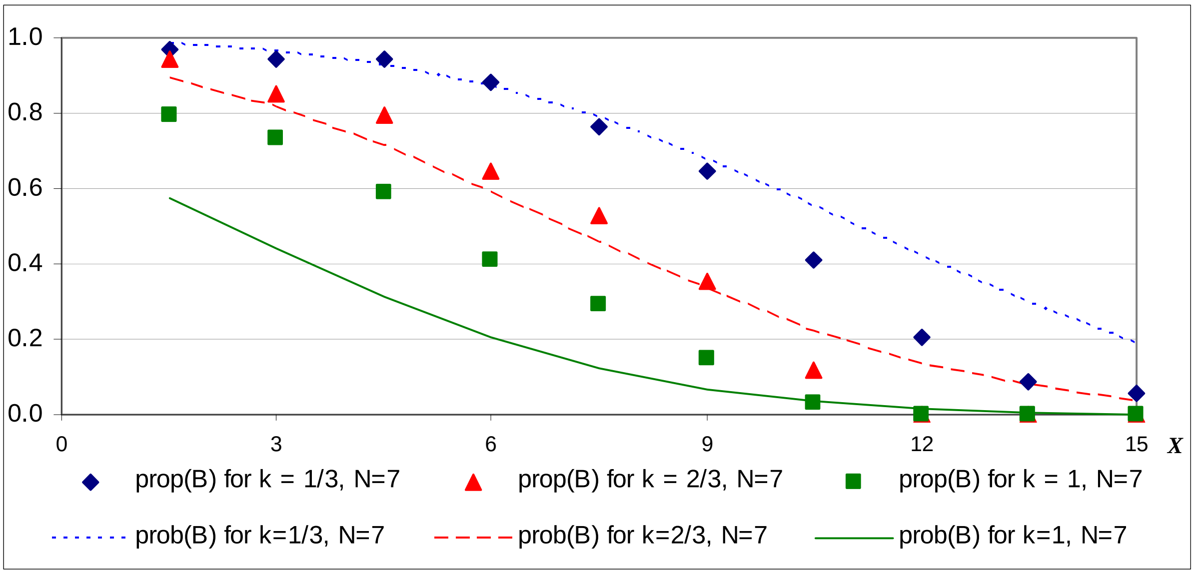
From an applied point of view, this means that by introducing a middle option with appropriate payoffs, the equilibrium selection can be turned from the inefficient to the efficient equilibrium of the original game. If the GGS is actually suited as a descriptive theory, the new middle action would not even be chosen. The mere existence of this option is sufficient for turning decisions around. The intuitive reason is that players are less afraid of the consequences of strategic uncertainty. In the original game players withdraw out of fear that others might do so. In the 3-action game, they do not withdraw, because they could better buy the insurance. However, if players buy an insurance anyway, the risk associated with rolling over is smaller and so they can as well go all the way.

Highly incomplete section to be merged with discussion of other experiments

Whether the GGS is a descriptive theory is ultimately an empirical question. In an experiment, Heinemann, Nagel, and Ockenfels (2009) provided a set of supermodular binary-choice games with multiple equilibria. These games had the structure that groups of N subjects simultaneously had to choose between two options A and B. The payoff for A was a fixed amount $X \leq 15$ Euros that varied between the different games. The payoff for B was 15 Euros, provided that at least K group members decided for B, and zero otherwise. The hurdle K was also varied between games. Different sessions used different group sizes and the total experiment spanned a range of 90 different coordination games. In these one-shot games, subject's choices were all over the place, but the proportion of subjects choosing B was clearly responsive to the parameters X and K. A higher hurdle for success of B or higher payoffs for A reduced the proportion of B-choices, which is in line with predictions of the GGS. The GGS in these games selects A if and only if

$$X > 15 \cdot \left(1 - \frac{K-1}{N}\right) .$$

Heinemann, Nagel, and Ockenfels (2009) used two versions of a global game with positive variances for describing the distribution of choices observed in the experiment. The noise distribution served as a tool for allowing subjects to take different actions and behave *as if* they had different signals about an underlying state parameter, while effectively they had perfect information about the games' payoffs. Fitting the variance of private signals to observations, they found a surprisingly good fit of actual observations and could apply the concept also for out-of sample predictions. Figure XX compares the observed proportion of B choices for various parameter combinations with the probability of a subject choosing B in the estimated global game, where k = (K-1)/(N-1). [elaborate: describe the experiment in a nutshell, present figure of a fitted global game, discuss comparative statics properties, quote Schmidt et al. ().]



We usually think of equilibrium concepts as being descriptive in repeated games, because equilibrium resembles the idea that chosen strategies are best responses to each other. Heinemann, Nagel, and Ockenfels (2009) also showed that the GGS is very close to the best response of what subjects actually did. From experiments on repeated coordination games, we know that they tend to follow best-response dynamics, that is, subjects coordinate on the strategy that is a best response to the first round(s). In that sense, the GGS can be used as a descriptive theory for repeated coordination games, at least, if they fall into the class tested by Heinemann, Nagel, and Ockenfels (2009)

For one-shot games, the GGS can obviously not be a descriptive theory. However, since it describes a best response, a single player who happens to be in a coordination game situation, may do as an individual recommendation given to a player who happens to be in a situation of a coordination game may do well following the recommendation provided by the GGS. In that sense, GGS can serve as a normative theory for individual players.

Note, however, that the GGS may select the inefficient equilibrium. So, it is certainly not a good recommendation for a social planner who wants to achieve efficiency, and, for example, avoid inefficient bank runs.

We do not know, whether these arguments would also hold for the 3-action game laid out in this section, as this game has not yet been tested in the lab.

Application 3: Asymmetric players

The third application is motivated by the problem of marketing a new network good and was inspired by Ruffle, Weiss, and Etziony (2010). The payoff to any agent buying a network good is increasing in the number of other agents buying the same good. In addition, payoffs do not need to be the same for all agents. The following payoff table is an example of such a market. Negative numbers mean that the payoff from the network good is lower than its price.

V(i, n)	number of adopters n											
Player i	1	2	3	4	5	6	7	8	9	10	11	12
A	-4	-1	2	5	8	11	14	17	20	23	26	29
B	-4	-1	2	5	8	11	14	17	20	23	26	29
C	-4	-1	2	5	8	11	14	17	20	23	26	29
D	-13	-10	-7	-4	-1	2	5	8	11	14	17	20
E	-13	-10	-7	-4	-1	2	5	8	11	14	17	20
F	-13	-10	-7	-4	-1	2	5	8	11	14	17	20
G	-25	-22	-19	-13	-10	-7	-4	-1	2	5	8	11
H	-25	-22	-19	-13	-10	-7	-4	-1	2	5	8	11
I	-25	-22	-19	-13	-10	-7	-4	-1	2	5	8	11
J	-34	-31	-28	-25	-22	-19	-13	-10	-7	-4	-1	2
K	-34	-31	-28	-25	-22	-19	-13	-10	-7	-4	-1	2
L	-34	-31	-28	-25	-22	-19	-13	-10	-7	-4	-1	2

Aquisition of a network good. There are M types of players with different payoff functions.

$v_i(n)$ = agent i‘s payoff from entry if n players enter in total.

Agents with the same payoff function belong to the same type.

Order the types s.t. „i belongs to a lower type than j“ iff $v_i(n) \geq v_j(n)$ for all n with at least one strict inequality.

Strategy combinations are partially ordered: $a \geq a'$ iff $a_i \geq a'_i$ for all i.

Define a^0 as the strategy combination, where everybody stays out,

a^1 as the strategy combination, where all players of type 1 enter, others stay out,

a^k as the strategy combination, where all players of types 1 to k enter and players of higher types stay out. => a^M = all players enter.

Look at restricted games with all strategies in $[a^{k-1}, a^k]$ for $k = 1, \ldots, M$.

In each of these restricted games, only players of type k have to decide. It is described by payoffs on the block diagonal.

It is a symmetric binary-action game between players of the same type.

It is noise independent and the GGS is given by the best response of a type-k player to a uniform distribution on the number of entrants among the other players of his own type (with players of lower types entering, higher types staying out).

Block diagonals are the same for all types:

V(i, n)	number of adopters n

Player i	1	2	3
A	-4	-1	2
B	-4	-1	2
C	-4	-1	2

The expected payoff given uniform distribution on the number of other players adopting is -1<0. Hence, in all restricted games, the lower strategy combination a^{k-1} is selected, and the GGS of the complete game is that no player adopts the good. Again, the GGS is noise independent.

Experiment: 2 sessions with 12 players each

Subjects were playing 20 different games in random order without feedback. Roles were randomly assigned to subjects for each game independently.

In each game, subjects could choose between two options:

For option A, they received 34 ECU.

Payoffs for option B were presented by payoff tables.

First, subjects had to answer comprehensive questions to make sure that they understood how to read the payoff tables.

Results (cf. Appendix A):

Low types tend to opt for B, high types tend to opt for A.

Comparative statics: higher payoffs in cells off the diagonal lead to more entries.

GGS-prediction of same behavior across types does not hold.

Behavior may be explained by global-game equilibrium with positive variance (cf. HNO 2009)

or by „naive" GGS: best response to uniform distribution on the number of others entering (entry if average number in a row >34).

or by levels of reasoning:
Level 0: e.b. enters with prob. 50%, level k best response to level k – 1.
In our games, level k = level 1 for k>1.

Appendix B …

References:

Angeletos, G.-M., and A. Pavan (2004), "Transparency of Information and Coordination in Economies with Investment Complementarities," *American Economic Review* (Papers and Proceedings), 94, 91-98.

Basteck, Christian, Tijmen Daniëls, and Frank Heinemann (2010). Characterising equilibrium selection in global games with strategic complementarities. SFB-649 working paper 2010-008.

Basteck, Christian, Tijmen Daniëls, and Frank Heinemann (2013) Characterising Equilibrium Selection in Global Games with Strategic Complementarities, *Journal of Economic Theory* 148, 2013, 2620-37.

Bryant, J. (1983) A simple rational-expectations Keynes-type model. Quart. J. Econ. 98 (1983) 525–528

Carlsson and van Damme (1993) Global games and equilibrium selection. Econometrica 61 (1993) 989–1018.

Corsetti, G., A. Dasgupta, S. Morris, and H. S. Shin. (2006) Does one Soros make a difference? a theory of currency crises with large and small traders. Rev. Econ. Stud. 71 (2004) 87–113.

Daniëls, Tijmen (2013) Note on Funding Risk and the Increase in Collateralised Lending, mimeo.

Diamond and Dybvig (1983)

Etziony and Weiss ()

Frankel, Morris, and Pauzner (2003) Equilibrium selection in global games with strategic complementarities. J. Econ. Theory 108 (2003) 1–44

Harsanyi and Selten (1988)

Heinemann, Frank, Rosemarie Nagel, and Peter Ockenfels (2009) Measuring strategic uncertainty in coordination games. Rev. Econ. Stud. 76 (2009) 181–221.

Morris, S. and H.S. Shin (2003) Global games: Theory and applications. In M. Dewatripont, L. P. Hansen, and S. J. Turnovsky, editors, Advances in Economics and Econometrics: Eight World Congress, p. 56–114. Cambridge University Press, Cambridge, 2003.

Ruffle, Bradley, Avi Weiss and Amir Etziony (2010), "Coordination and Critical Mass in a Network Market: An Experimental Evaluation," Working paper available at http://ssrn.com/abstract=1550723

Van Huyck, Battalio, and Beil (1990) Tacit coordination games, strategic uncertainty, and coordination failure. Amer. Econ. Rev. 80 (1990) 234–248.

Appendix A. Experiment on the game in Application 3

Payoff tables and results, comparison with theory predictions: GGS, estimated global game, level-k.

Appendix B. Estimating the global-game equilibrium in the game of Application 3

The net payoff for entry by agent *i* given that a total of *n* agents are entering is $v_i(n)$. Extend this by a state variable, s.t. payoff for entry is $v_i(n)+\theta$. Assume that the state variable has an improper uniform distribution on the reals (=> dominance regions exist). Agents receive i.i.d. private signals $x_i \sim N(\theta,\sigma^2)$. **A global-game equilibrium is a vector of thresholds**, s.t.
(1) each agent enters [does not enter] if his signal exceeds [falls short of] the threshold, and
(2) an agent receiving the threshold signal is indifferent.

Players $i \in \{1,...,N\}$, denote the type of player *i* by $k(i)$. In equilibrium, players of the same type have the same threshold. Denote the equilibrium threshold of type k by x^k. We have $N=12$. The number of types varies over 1, 2, 4, and 12.

In our games, we can order types such that

$k(i)<k(j)$ if and only if $v_i(n) \geq v_j(n)$ for all n, with at least one strict inequality.

So players ABC belong to type 1, players DEF are type 2, and so on, in the example above.

Due to the order of types, $x^k \leq x^{k+1}$.

(1) Give that the true realization of $\theta=0$, the probability that player j enters is

$$prob(x_j \geq x^{k(j)}) = 1-\Phi\left(x^{k(j)}/\sigma\right) \tag{1}$$

with Φ denoting the cumulative standard normal distribution. For any given value of sigma, this is the probability for a subject entering that we use in the Maximum-likelihood estimate. Before we can do so, we need to find the thresholds that are associated with a particular sigma. This comes from condition (2).

The first step is thus to find the equilibrium thresholds for any sigma. The second step is then to find the sigma that maximizes the likelihood of our observations.

(2) An arbitrary agent i is indifferent at signal $x_i = x^{k(i)}$, iff

$$E(v_i(n)+\theta \mid x_i)=0. \tag{2}$$

$$E(v_i(n) \mid x_i) = \int_{-\infty}^{\infty} \left[\sum_{n=1}^{N} prob(n-1\, other\ agents\ enter \mid \theta)\cdot v_i(n) \right] \cdot f(\theta \mid x_i)\, d\theta \tag{3}$$

and $E(\theta \mid x_i) = x_i$, where $f(\theta \mid x_i) = \phi\left(\dfrac{\theta - x_i}{\sigma}\right)$ and ϕ is the non-cumulative standard normal distribution. Note that we can reformulate

$$E(v_i(n) \mid x_i) = \sum_{n=1}^{N} v_i(n) \int_{-\infty}^{\infty} prob(n-1\, other\ agents\ enter \mid \theta)\cdot f(\theta \mid x_i)\, d\theta\ . \tag{4}$$

Denote the conditional probability that $(n - 1)$ other subjects enter conditional on signal x_i by

$$\hat{p}^{-i}(n-1|x_i) = \int_{-\infty}^{\infty} prob(n-1\, other\ agents\ than\ i\ enter\,|\,\theta)\cdot f(\theta|x_i)d\theta\,. \qquad (5)$$

The tricky part is to describe this probability.

Some helpful notation: Denote the **conditional probability that another agent of type *k* enters given state** θ by $p_\theta^k = prob(x_j \geq x^{k(j)}\,|\,\theta) = 1-\Phi\left(\frac{x^{k(j)}-\theta}{\sigma}\right)$, (6)

and the conditional probability that *m* agents of *other* types than $k(i)$ enter, for a given state θ, by $\tilde{p}^{-i}(m|\theta)$.

12 types with 1 agent each (Games 17-20): Note that the other games are special cases with subjects of the same type having the same threshold in equilibrium. So, I do not lay out those games explicitly.

Games 17-20 are actually dominance solvable and have a unique equilibrium. The global game may yield a better description of behaviour anyway. It basically accounts for strategic uncertainty in a game where deduction should eliminate this uncertainty. We know, however, from observations that people are uncertain and do not put probability 1 on others' rationality, leave alone higher-order rationality. The logit equilibrium captures this already (see also Kübler & Weizsäcker, 2004).

So, let us continue the assumption that players are uncertain about which game they are playing (that is theta is uncertain).

Claim 1: An equilibrium can be described by a vector of thresholds $x^k \leq x^{k+1}$.

Then, we can continue to use the indifference condition

$$E(v_i(n)|x_i) = \sum_{n=1}^{N} v_i(n)\cdot \int_{-\infty}^{\infty} prob(n-1\, other\ agents\ enter\,|\,\theta) f(\theta|x_i)d\theta = \sum_{n=1}^{N} v_i(n)\cdot\hat{p}^{-i}(n-1|x_i) = -x_i$$

and

$$prob(n-1\, other\ agents\ enter\,|\,\theta)$$
$$= \tilde{p}^{-i}(n-1|\theta) = prob(n-1\ \ agents\ \ from\ other\ types\ than\ i\ enter\,|\,\theta)$$

= probability that n-1 of the other signals are below the individual thresholds x^k, $k \neq k(i)$.

For each agent (of another type), this probability is given by p_θ^k. The probability $\tilde{p}^{-i}(n-1|\theta)$ is the solution to a combinatorial problem.

Basically, any combination of n-1 other agents must be accounted for with the probability that these n-1 agents enter. The latter is the product of p_θ^k for the n-1 different agents k. So,

$\widetilde{p}^{-i}(n-1\,|\,\theta)$ is the sum_(over all possible combinations of n-1 other agents) of the products of these agents' entry probabilities,

$$\widetilde{p}^{-i}(n-1\,|\,\theta)=\sum_{\textit{all combinations of } n-1 \textit{ other agents}}\left(\prod_{k\in combination} p_\theta^k\cdot\prod_{k\notin combination}(1-p_\theta^k)\right).$$

Let me here use the letters i, j, k, … for agents and types (since each agent is one type).

$$\widetilde{p}^{-i}(0\,|\,\theta)=\prod_{k\neq i}(1-p_\theta^k),\qquad \widetilde{p}^{-i}(1\,|\,\theta)=\sum_{k\neq i}\left(p_\theta^k\cdot\prod_{\substack{k'\neq i\\ k'\neq k}}(1-p_\theta^{k'})\right),$$

Define the set of players excluding *i*, *k*, *k*', and so on by
$K-\{i,k,k'\}=\{j\,|\,j\neq i\wedge j\neq k\wedge j\neq k'\}$. Then,

$$\widetilde{p}^{-i}(2\,|\,\theta)=\sum_{k\neq i}\left(p_\theta^k\cdot\sum_{\substack{k'>k,\\ k'\neq i}}\left(p_\theta^{k'}\cdot\prod_{j\in K-\{i,k,k'\}}(1-p_\theta^j)\right)\right),$$

$$\widetilde{p}^{-i}(3\,|\,\theta)=\sum_{k\neq i}\left(p_\theta^k\cdot\sum_{\substack{k'>k,\\ k'\neq i}}\left(p_\theta^{k'}\cdot\sum_{\substack{k''>k',\\ k''\neq i}}\left(p_\theta^{k''}\cdot\prod_{j\in K-\{i,k,k',k''\}}(1-p_\theta^j)\right)\right)\right),$$

$$\widetilde{p}^{-i}(4\,|\,\theta)=\sum_{k\neq i}\left(p_\theta^k\cdot\sum_{\substack{k'>k,\\ k'\neq i}}\left(p_\theta^{k'}\cdot\sum_{\substack{k''>k',\\ k''\neq i}}\left(p_\theta^{k''}\cdot\sum_{\substack{k'''>k'',\\ k'''\neq i}}\left(p_\theta^{k'''}\cdot\prod_{j\in K-\{i,k,k',k'',k'''\}}(1-p_\theta^j)\right)\right)\right)\right),$$

$$\widetilde{p}^{-i}(5\,|\,\theta)=\sum_{k\neq i}\left(p_\theta^k\cdot\sum_{\substack{k'>k,\\ k'\neq i}}\left(p_\theta^{k'}\cdot\sum_{\substack{k''>k',\\ k''\neq i}}\left(p_\theta^{k''}\cdot\sum_{\substack{k'''>k'',\\ k'''\neq i}}\left(p_\theta^{k'''}\cdot[\sum_{\substack{k''''>k''',\\ k''''\neq i}}p_\theta^{k''''}\prod_{j\in K-\{i,k,k',k'',k''',k''''\}}(1-p_\theta^j)]\right)\right)\right)\right),$$

$$\widetilde{p}^{-i}(6\,|\,\theta)=$$

$$\sum_{k\neq i}\left((1-p_\theta^k)\cdot\sum_{\substack{k'>k,\\ k'\neq i}}\left((1-p_\theta^{k'})\cdot\sum_{\substack{k''>k',\\ k''\neq i}}\left((1-p_\theta^{k''})\cdot\sum_{\substack{k'''>k'',\\ k'''\neq i}}\left((1-p_\theta^{k'''})\cdot[\sum_{\substack{k''''>k''',\\ k''''\neq i}}(1-p_\theta^{k''''})\prod_{j\in K-\{i,k,k',k'',k''',k''''\}}p_\theta^j\ ]\right)\right)\right)\right),$$

$$\widetilde{p}^{-i}(7\,|\,\theta)=\sum_{k\neq i}\left((1-p_\theta^k)\cdot\sum_{\substack{k'>k,\\ k'\neq i}}\left((1-p_\theta^{k'})\cdot\sum_{\substack{k''>k',\\ k''\neq i}}\left((1-p_\theta^{k''})\cdot\sum_{\substack{k'''>k'',\\ k'''\neq i}}\left((1-p_\theta^{k'''})\cdot\prod_{j\in K-\{i,k,k',k'',k'''\}}p_\theta^j\right)\right)\right)\right),$$

$$\widetilde{p}^{-i}(8\,|\,\theta)=\sum_{k\neq i}\left((1-p_\theta^k)\cdot\sum_{\substack{k'>k,\\ k'\neq i}}\left((1-p_\theta^{k'})\cdot\sum_{\substack{k''>k',\\ k''\neq i}}\left((1-p_\theta^{k''})\cdot\prod_{j\in K-\{i,k,k',k''\}}p_\theta^j\right)\right)\right),$$

$$\widetilde{p}^{-i}(9\,|\,\theta)=\sum_{k\neq i}\left((1-p_\theta^k)\cdot\sum_{\substack{k'>k,\\ k'\neq i}}\left((1-p_\theta^{k'})\cdot\prod_{j\in K-\{i,k,k'\}}p_\theta^j\right)\right),$$

$$\widetilde{p}^{-i}(10\,|\,\theta)=\sum_{k\neq i}\left((1-p_\theta^k)\cdot\prod_{\substack{k'\neq i\\ k'\neq k}}p_\theta^{k'}\right),$$ and $$\widetilde{p}^{-i}(11\,|\,\theta)=\prod_{k\neq i}p_\theta^k\,.$$